Notes on "Residual Seasonality in Monthly Core Inflation"*

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Contents

*The views expressed herein are my own and do not necessarily represent the views of the Federal Reserve Bank of Richmond or the Federal Reserve System. I would like to thank Jonathan Wright for making his code to test for seasonality available to me and David Ramachandran for research assistance. Andreas Hornstein: andreas.hornstein@rich.frb.org

Abstract

This note provides background for the Economic Brief "Residual Seasonality in Monthly Core Inflation"

1 Some plots

1.1 Core CPI, NSA and SA, monthly inflation rates

The following two figures plot the difference between monthly Core CPI inflation (at an annual rate) and its symmetric 13-month moving average for not seasonally adjusted (NSA) inflation, Figure 1, and seasonally adjusted (SA) inflation, Figure 2. Each panel plots the difference for a month, for example, the panel labelled 'January' plots the difference for every January in the years 1960 to 2023.

Figure 1: Seasonality in Monthly Core CPI (NSA) Inflation, 1960-2023

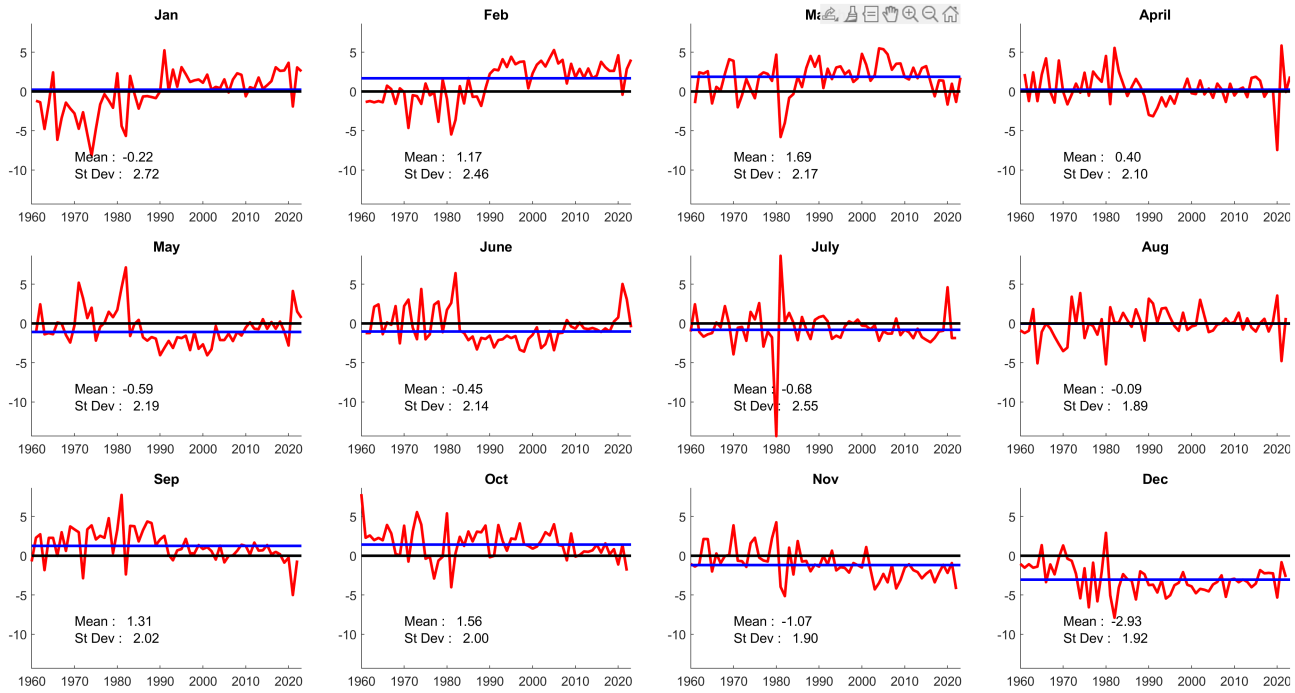
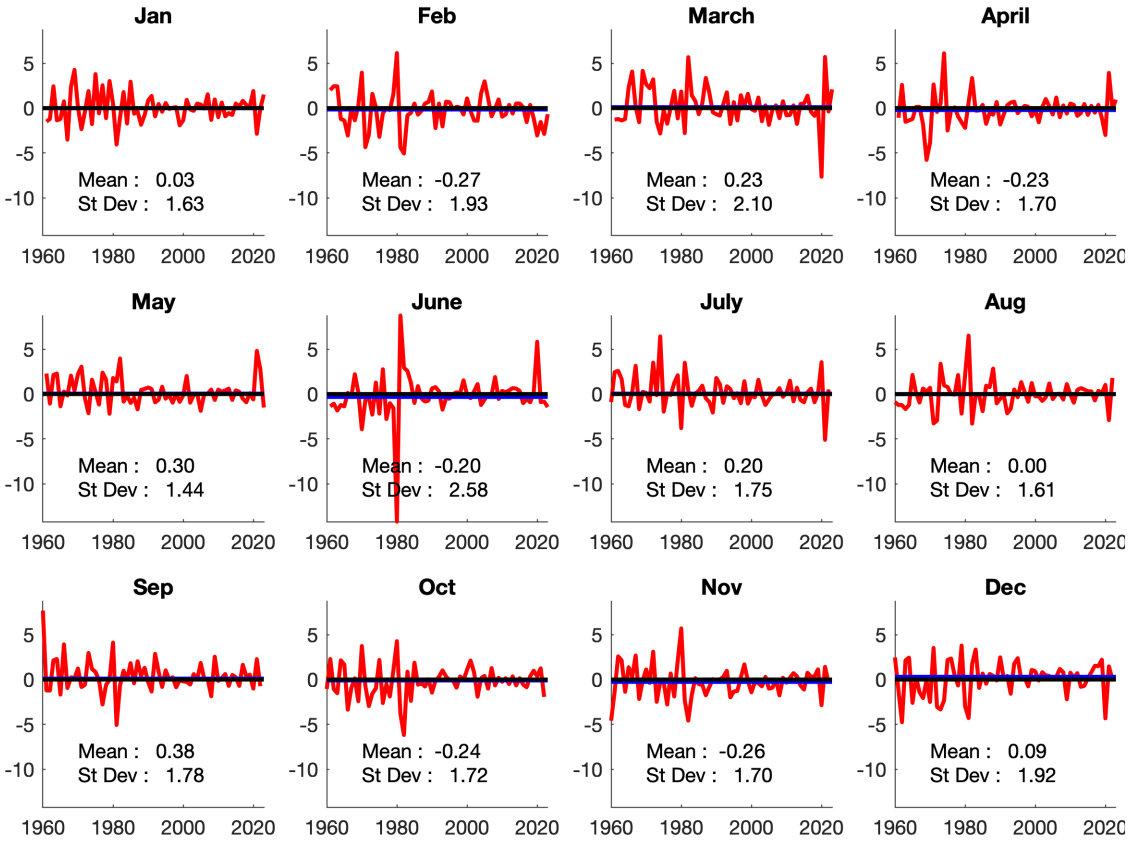


Figure 2: Residual Seasonality in Monthly Core CPI (SA) Inflation, 1960-2023



1.2 Core PCE Inflation, SA, monthly and quarterly

The following two figures plot the difference between monthly (quarterly) Core PCE inflation (at an annual rate) and its symmetric 13-month (5-quarter) moving average for seasonally adjusted (SA) inflation, Figure 3 and Figure 4 respectively. Each panel plots the difference for a month (quarter), for example, the panel labelled 'January' in Figure 3 plots the difference for every January in the years 1960 to 2023, and the panel labelled 'Jan-March' in Figure 4 plots the difference for every first quarter in the years 1960 to 2023

Figure 3: Residual Seasonality in Monthly Core PCE Inflation, 1960-2023

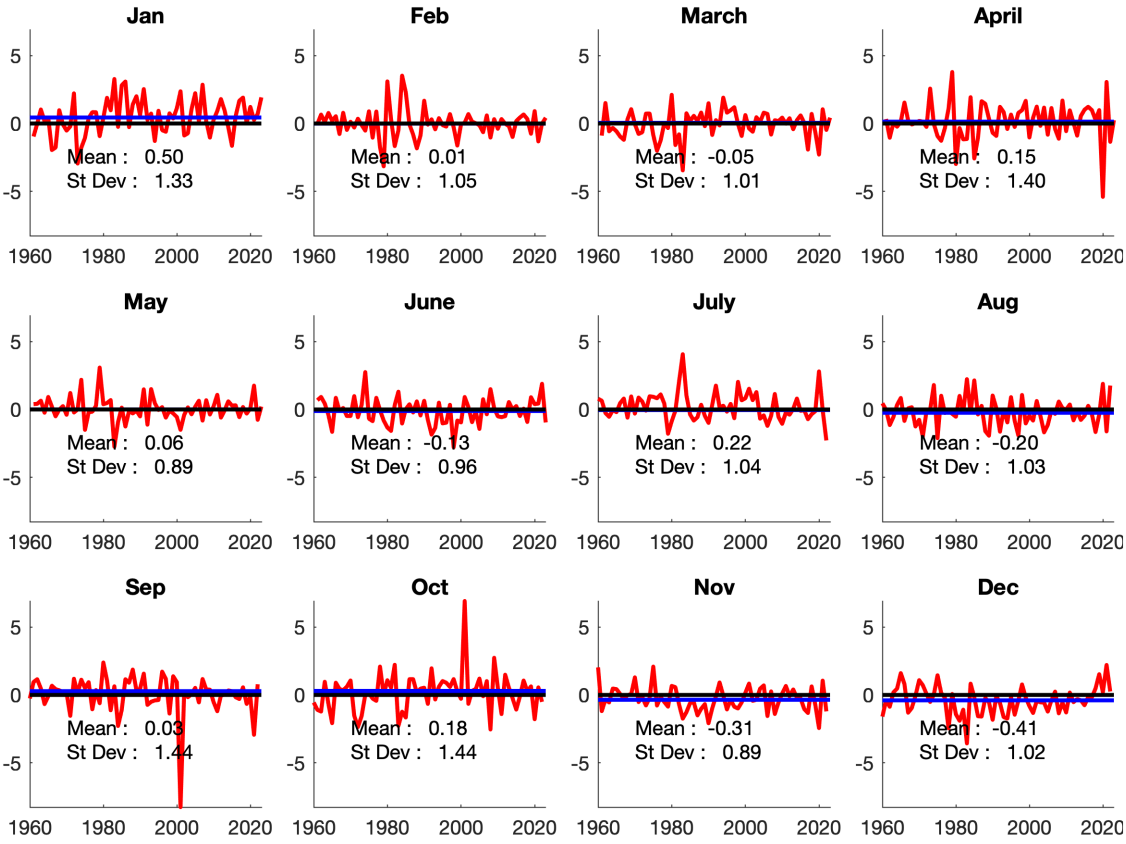
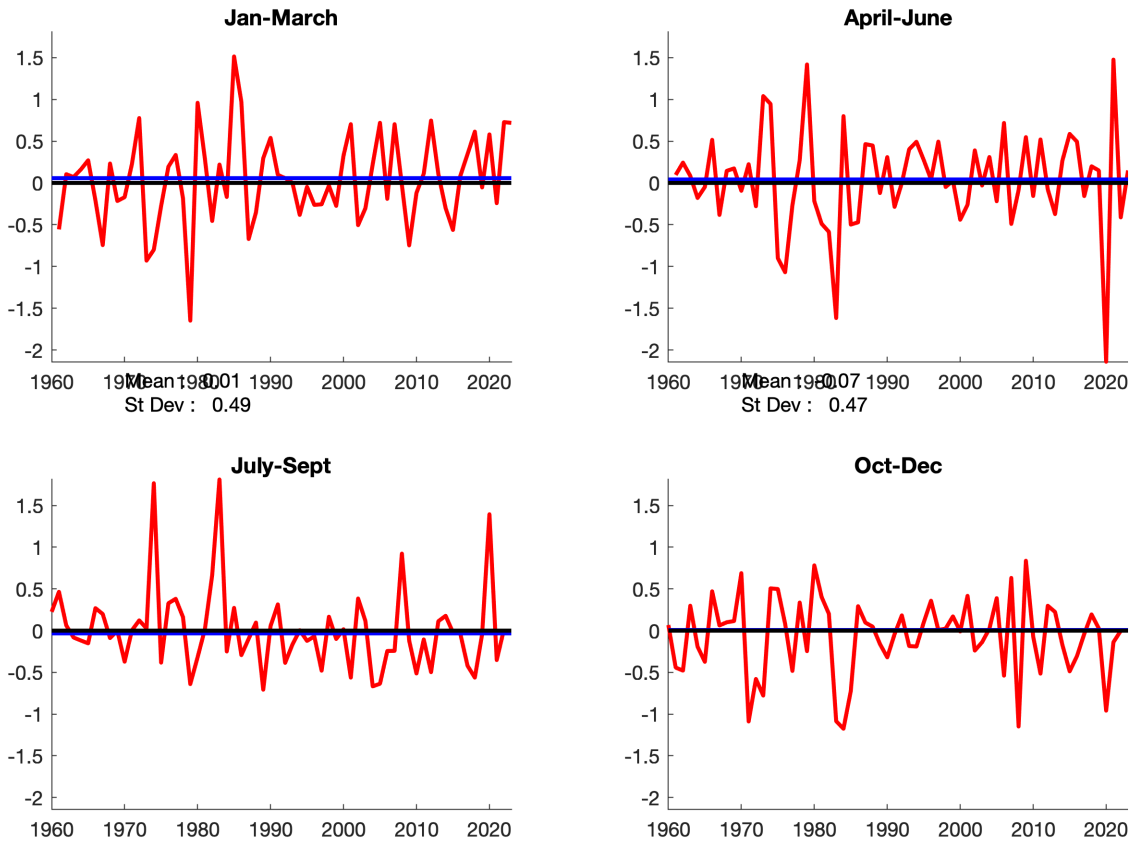


Figure 4: Residual Seasonality in Quarterly Core CPE Inflation, 1960-2023



1.3 Monthly nonfarm payroll employment changes, NSA and SA

The following two figures plot the monthly change in nonfarm payroll employment (in thousands) for not seasonally adjusted (NSA) employment, Figure 5, and seasonally adjusted (SA) employment, Figure 6. Each panel plots the employment change for a month. For example, the panel labeled 'January' plots the employment change for every January in the years 2000 to 2023. Since the COVID-induced employment changes were exceptionally large, the means are calculated excluding 2020. Note that there is no apparent residual seasonality in the monthly payroll changes (SA), rather a positive increment in every month, reflecting the growth in payroll employment over time.

Figure 5: Seasonality in Monthly Changes of Payroll Employment (NSA), 2000-2023

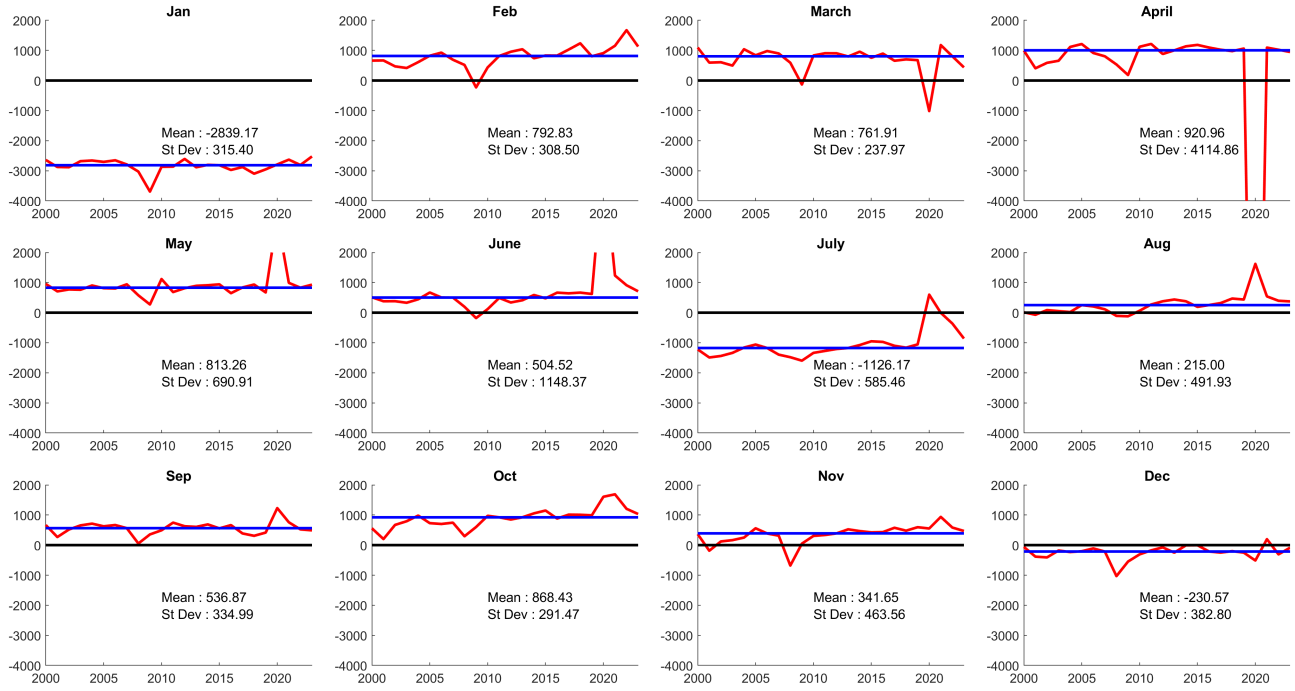
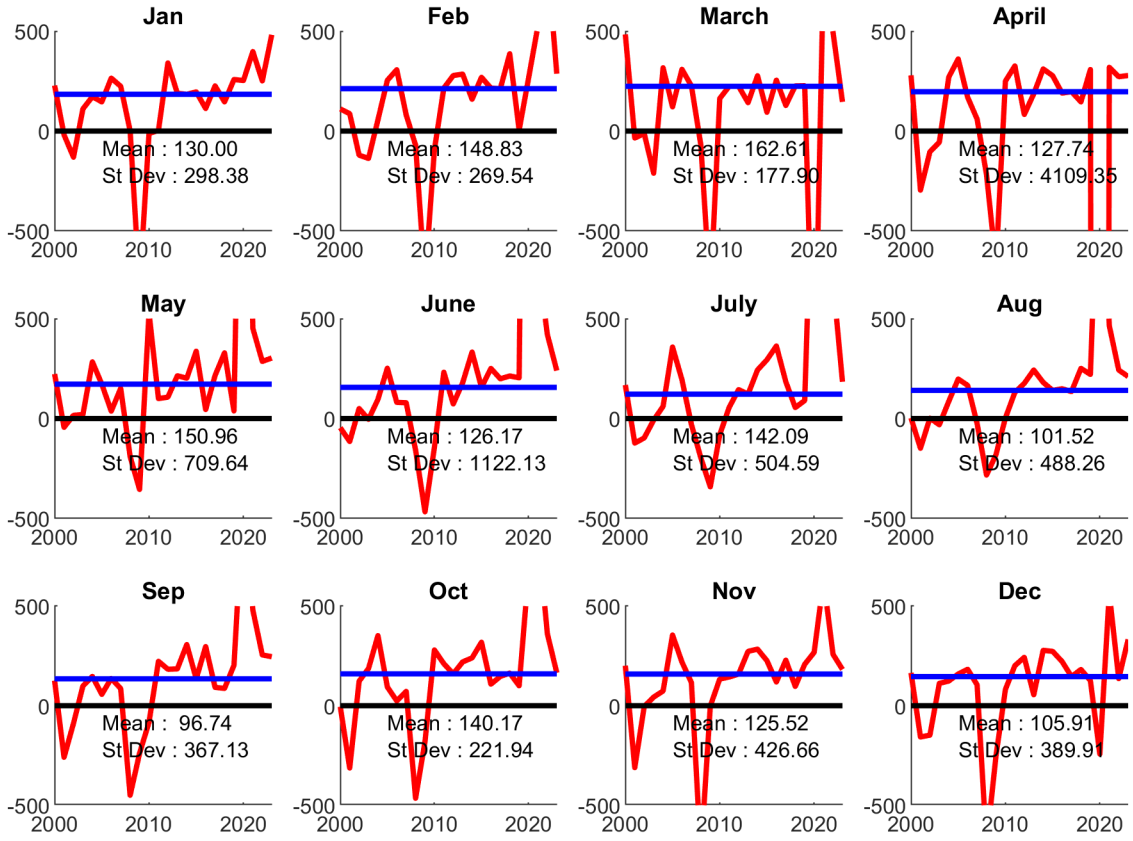


Figure 6: Residual Seasonality in Monthly Changes of Payroll Employment (SA), 2000-2023



2 Testing for seasonality

This section describes the Wald-test for stable seasonals, as well as the test for stable seasonality from Canova and Hansen (1995) as implemented by Wright (2018).

2.1 Wald-test for stable seasonals

For the time series $\{y_t\}$ with n_s observations per year run the regression

$$y_t = \alpha + \rho y_{t-1} + \sum_{i=1}^{n_s-1} \beta_i s_{i,t} + u_t \text{ with } E[u_t] = 0 \text{ for } t = 1, \ldots, T \tag{1}$$

where $s_{i,t} = 1$ when t is the i-th period of the year and 0 otherwise. Note that the seasonal effects, β_i are additive and constant across the years.

The regression is estimated with HAC robust estimates for the error term with a lag-truncation parameter of $1.3\sqrt{T}$. The Wald-test statistic W is for $H_0 : \beta_1 = \ldots = \beta_{n_s-1} = 0$. The distribution of the Wald-test statistic is based on Kiefer and Vogelsang (2005). For more details see Wright (2018).

2.2 CH-test for unstable seasonality

For equation (1) replace the constant term with the remaining seasonal. Canova and Hansen (1995) consider the possibility that the coefficients on the seasonal factors follow a random walk

$$y_t = \rho y_{t-1} + \sum_{i=1}^{n_s} \beta_i s_{i,t} + u_t \qquad \text{with } E[u_t] = 0 \text{ for } t = 1, \ldots, T \tag{2}$$

$$\beta_{i,t} = \beta_{i,t-1} + e_{i,t} \qquad \text{with } E[e_{i,t}] = 0 \text{ for } t = 1, \ldots, T \text{ and } i = 1, \ldots, n_s \tag{3}$$

with covariance matrix for the error terms

$$E\left[e_t e_t'\right] = \tau^2 V \tag{4}$$

The CH-test statistic for unstable seasonals L is for $H_0 : \tau^2 = 0$. The distribution L^* of the CH-test is based on Cho and Vogelsang (2017). For more details see Wright (2018).

Canova and Hansen (1995), section 3.3, note that for this specification the joint hypothesis H_0 not only tests for a unit root in the seasonal coefficients, but also at frequency zero. They propose to renormalize the seasonal dummies β in terms of a constant term, μ, and $n_s - 1$ seasonal dummies, η,

$$\beta = \iota_{n_s}\mu + D\eta \text{ with } D = \begin{bmatrix} I_{n_s-1} \\ -\iota'_{n_s-1} \end{bmatrix}$$

You now include the constant term in the seasonal regression

$$y_t = \rho y_{t-1} + \mu + \sum_{i=1}^{n_s-1} \beta_i s^*_{i,t} + u_t \text{ for } t = 1, \ldots, T$$

but for the modified seasonals

$$s^* = D's$$

2.3 Joint test for unstable seasonality

The statistic for the joint test that the seasonals are stable and zero, that is, for $H_0 : \beta_1 = \ldots = \beta_{n_s-1} = 0$ and $\tau^2 = 0$, is the sum of the Wald-statistic and the CH-statistic, $W + L$. The distribution of this test statistic is given by the random variable $L^* + L(1)^*$. For more details see Wright (2018).

3 EB tables

3.1 Seasonality in Core CPI and Core PCE

Table 1: Monthly Seasonality in Core Price Inflation, 2000-2023

Month	Core CPI (NSA)		Core CPI (SA)		Core PCE (SA)	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Jan	1.35	1.36	0.10	1.06	0.72	1.19
Feb	2.97	1.27	-0.38	1.38	-0.02	0.60
March	2.16	1.86	-0.19	2.25	-0.05	0.82
April	0.25	2.22	-0.05	1.28	0.19	1.60
May	-0.77	1.71	0.21	1.42	0.03	0.69
June	-0.45	1.77	0.14	1.48	0.05	0.82
July	-0.95	1.42	-0.08	1.50	0.02	1.06
Aug	0.04	1.57	0.02	0.94	-0.24	0.92
Sep	0.20	1.35	0.23	1.06	-0.40	1.91
Oct	1.00	1.34	0.05	1.00	0.39	1.82
Nov	-2.24	1.27	-0.27	0.98	-0.28	0.90
Dec	-3.35	1.12	0.26	1.36	-0.25	0.86

Notes: Mean and standard deviation of monthly inflation for Core CPI (NSA and SA) and Core PCE (SA).

3.2 Wald-test for seasonality in Core CPI (NSA and SA) and Core PCE

$$H_0\text{: } \beta_i = 0 \text{ for } i = 1, \ldots, n_s - 1$$

Table 2: F-test for stable seasonals: test statistic

Sub-sample	Core CPI (NSA)	Core CPI (SA)	Core PCE (SA)
1960-1979	180.67	28.64	36.77
1980-1999	270.89	11.54	136.62
2000-2023	1027.15	20.26	38.74

Table 3: F-test for stable seasonals: p-values

Sub-sample	Core CPI (NSA)	Core CPI (SA)	Core PCE (SA)
1960-1979	0.00	0.32	0.17
1980-1999	0.00	0.86	0.00
2000-2023	0.00	0.51	0.12

3.3 CH-test for seasonality in Core CPI and Core PCE

$$H_0\text{: } \tau^2 = 0$$

Table 4: CH-test for unstable seasonals: test statistic

Sub-sample	Core CPI (NSA)	Core CPI (SA)	Core PCE (SA)
1960-1979	1.59	1.44	1.39
1980-1999	1.66	1.25	1.50
2000-2023	1.94	1.60	1.63

Table 5: CH-test for unstable seasonals: p-values

Sub-sample	Core CPI (NSA)	Core CPI (SA)	Core PCE (SA)
1960-1979	0.37	0.70	0.79
1980-1999	0.23	0.94	0.60
2000-2023	0.00	0.40	0.32

3.4 CHJ-test for seasonality in Core CPI and Core PCE

$$H_0\text{: } \tau^2 = 0 \text{ and } \beta_i = 0 \text{ for } i = 1, \ldots, n_s - 1$$

Table 6: CHJ-test for non-zero unstable seasonals: test statistic

Sub-sample	Core CPI (NSA)	Core CPI (SA)	Core PCE (SA)
1960-1979	182.27	30.08	38.16
1980-1999	272.55	12.79	138.11
2000-2023	1029.09	21.86	40.37

Table 7: CHJ-test for non-zero unstable seasonals: p-values

Sub-sample	Core CPI (NSA)	Core CPI (SA)	Core PCE (SA)
1960-1979	0.00	0.32	0.17
1980-1999	0.00	0.87	0.00
2000-2023	0.00	0.51	0.12

4 Additional tables

Core CPI (SA)

- There is no evidence of residual seasonality in monthly Core CPI (SA) inflation over 20-year rolling windows for sub-samples of currently available SA data. See Table 8.
- SA at the end of the sample is frequently revised: Symmetric MA filters are calculated using forecasts of the NSA inflation rates. However, there is no evidence for residual seasonality in monthly Core CPI inflation over 20-year rolling windows for sub-samples from real-time SA data. See Table 9. Furthermore, real-time data is not associated with uniformly lower p-values for residual seasonality.
- Monthly CPI inflation is very noisy, which might hide underlying seasonality at quarterly frequencies. For this to matter, the monthly error terms would have to be correlated in a very particular way, within a quarter. Furthermore, our seasonal regressions account for general autocorrelation in the error terms. Having said this, there is still no evidence of residual seasonality in quarterly Core CPI inflation over 20-year rolling windows for sub-samples from real-time SA data. See Table 10.
- Alternatively, we calculate quarterly inflation rates of the Core CPI index as 3-month averages of monthly inflation rates. Again, there is no evidence of residual seasonality. See Table 11.

Core PCE (SA)

- There is strong evidence of residual seasonality in monthly Core PCE (SA) inflation over 20-year rolling windows for sub-samples of currently available SA data. There is no evidence of unstable residual seasonality, as the p-values for the CH-statistic are all quite high. See Table 12. The evidence of residual seasonality is weaker for 10-year windows than for the 20-year window. See Table 13. Thus residual seasonality is mostly for the pre-2008 period. Overall, it is surprising to find residual seasonality in monthly Core PCE inflation even though the BEA revised the SA procedures for all NIA and finds no residual seasonality in the quarterly series of the major NIA aggregates, Cowan et al. (2018).
- There is almost no evidence of residual seasonality in quarterly Core PCE (SA) inflation, consistent with Cowan et al. (2018). Furthermore, the evidence for residual seasonality is limited to the real-time data series prior to 2013.

4.1 Residual seasonality in Core CPI (SA)

Table 8: p-values for Monthly Inflation, Current Data

Last Year	F-test	CH-test	CHJ-test
2000 :	0.755	0.730	0.757
2001 :	0.528	0.540	0.529
2002 :	0.474	0.663	0.476
2003 :	0.624	0.905	0.631
2004 :	0.602	0.958	0.614
2005 :	0.545	0.884	0.552
2006 :	0.587	0.880	0.594
2007 :	0.681	0.937	0.692
2008 :	0.601	0.945	0.610
2009 :	0.678	0.822	0.684
2010 :	0.658	0.962	0.670
2011 :	0.719	0.790	0.723
2012 :	0.709	0.667	0.712
2013 :	0.611	0.622	0.614
2014 :	0.538	0.732	0.542
2015 :	0.511	0.944	0.522
2016 :	0.833	0.954	0.843
2017 :	0.674	0.907	0.684
2018 :	0.384	0.885	0.390
2019 :	0.357	0.573	0.358
2020 :	0.441	0.840	0.446
2021 :	0.564	0.627	0.564
2022 :	0.522	0.651	0.523
2023 :	0.695	0.372	0.692

Notes: p-values for F-test, CH-test, and joint F&CH-test for rolling 20-year windows ending in last year (inclusive December). Using current data for monthly Core CPI (SA) inflation.

Table 9: p-values for Monthly Inflation, Current and Real-Time Data

Last year	Real time data	Current data
1998 :	0.750	0.857
1999 :	0.816	0.739
2000 :	0.608	0.706
2001 :	0.530	0.329
2002 :	0.234	0.304
2003 :	0.523	0.412
2004 :	0.542	0.441
2005 :	0.290	0.194
2006 :	0.208	0.317
2007 :	0.665	0.403
2008 :	0.188	0.407
2009 :	0.437	0.461
2010 :	0.643	0.802
2011 :	0.335	0.683
2012 :	0.543	0.721
2013 :	0.609	0.686
2014 :	0.530	0.615
2015 :	0.367	0.580
2016 :	0.680	0.880
2017 :	0.591	0.839
2018 :	0.215	0.651
2019 :	0.215	0.462
2020 :	0.486	0.566
2021 :	0.730	0.752
2022 :	0.642	0.709
2023 :	0.919	0.821

Notes: p-values for joint F&CH-test for rolling 20-year windows ending in last year (inclusive December). Using real-time data and currently available data.

Table 10: p-values for Quarterly Inflation, Current and Real-Time Data

Last year	Real time data	Current data
1998 :	0.770	0.925
1999 :	0.605	0.834
2000 :	0.698	0.896
2001 :	0.185	0.608
2002 :	0.615	0.869
2003 :	0.641	0.488
2004 :	0.576	0.429
2005 :	0.615	0.424
2006 :	0.744	0.617
2007 :	0.741	0.389
2008 :	0.422	0.353
2009 :	0.474	0.302
2010 :	0.970	0.614
2011 :	0.821	0.856
2012 :	0.810	0.925
2013 :	0.897	0.719
2014 :	0.912	0.859
2015 :	0.962	0.962
2016 :	0.768	0.999
2017 :	0.611	0.952
2018 :	0.708	0.905
2019 :	0.682	0.868
2020 :	0.657	0.747
2021 :	0.920	0.790
2022 :	0.868	0.889
2023 :	0.972	0.914

Notes: p-values for joint F&CH-test for rolling 20-year windows ending in last year (inclusive 4th quarter). Using real-time data and currently available data.

Table 11: p-values for 3-Month Averages of Monthly Inflation, Current and Real-Time Data

Last year	Real time data	Current data
1998 :	0.917	0.949
1999 :	0.840	0.881
2000 :	0.368	0.654
2001 :	0.374	0.575
2002 :	0.468	0.851
2003 :	0.920	0.935
2004 :	0.798	0.904
2005 :	0.668	0.828
2006 :	0.738	0.823
2007 :	0.861	0.722
2008 :	0.754	0.720
2009 :	0.792	0.643
2010 :	0.878	0.836
2011 :	0.604	0.894
2012 :	0.625	0.894
2013 :	0.628	0.307
2014 :	0.684	0.351
2015 :	0.909	0.464
2016 :	0.942	0.818
2017 :	0.209	0.572
2018 :	0.154	0.353
2019 :	0.177	0.313
2020 :	0.479	0.465
2021 :	0.956	0.878
2022 :	0.638	0.965
2023 :	0.989	0.968

Notes: p-values for joint F&CH-test for rolling 20-year windows ending in last year (inclusive 4th quarter). Using real-time data and currently available data.

4.2 Residual seasonality in Core PCE (SA)

Table 12: p-values for Monthly Inflation, Current Data, 20-Year Windows

Last Year	F-test	CH-test	CHJ-test
2000 :	0.000	0.409	0.000
2001 :	0.000	0.556	0.000
2002 :	0.000	0.736	0.000
2003 :	0.000	0.515	0.000
2004 :	0.000	0.802	0.000
2005 :	0.000	0.832	0.000
2006 :	0.000	0.875	0.000
2007 :	0.000	0.563	0.000
2008 :	0.000	0.790	0.000
2009 :	0.000	0.453	0.000
2010 :	0.000	0.404	0.000
2011 :	0.003	0.234	0.003
2012 :	0.000	0.537	0.000
2013 :	0.001	0.219	0.001
2014 :	0.000	0.479	0.000
2015 :	0.003	0.402	0.003
2016 :	0.003	0.593	0.003
2017 :	0.004	0.238	0.004
2018 :	0.006	0.197	0.006
2019 :	0.012	0.190	0.012
2020 :	0.047	0.239	0.046
2021 :	0.328	0.471	0.327
2022 :	0.470	0.411	0.469
2023 :	0.266	0.581	0.267

Notes: p-values for F-test, CH-test, and joint F&CH-test for rolling 20-year windows ending in last year (inclusive December). Using current data for monthly Core PCE (SA) inflation.

Table 13: p-values for Monthly Inflation, Current Data, 10-Year Windows

Last Year	F-test	CH-test	CHJ-test
2000 :	0.001	0.837	0.001
2001 :	0.008	0.591	0.008
2002 :	0.000	0.551	0.000
2003 :	0.001	0.299	0.001
2004 :	0.010	0.334	0.010
2005 :	0.001	0.457	0.001
2006 :	0.002	0.751	0.002
2007 :	0.010	0.293	0.010
2008 :	0.181	0.318	0.181
2009 :	0.146	0.677	0.146
2010 :	0.146	0.818	0.146
2011 :	0.461	0.533	0.461
2012 :	0.041	0.976	0.042
2013 :	0.028	0.772	0.028
2014 :	0.019	0.893	0.019
2015 :	0.103	0.813	0.104
2016 :	0.235	0.893	0.237
2017 :	0.148	0.358	0.147
2018 :	0.154	0.253	0.154
2019 :	0.272	0.038	0.270
2020 :	0.450	0.109	0.448
2021 :	0.482	0.425	0.481
2022 :	0.840	0.743	0.842
2023 :	0.670	0.850	0.672

Notes: p-values for F-test, CH-test, and joint F&CH-test for rolling 10-year windows ending in last year (inclusive December). Using current data for monthly Core PCE (SA) inflation.

Table 14: p-values for Quarterly Inflation, Current and Real-Time Data, 20-Year Windows

Last year	Real time data	Current data
1998 :	0.261	0.889
1999 :	0.513	0.956
2000 :	0.390	0.923
2001 :	0.231	0.781
2002 :	0.032	0.704
2003 :	0.342	0.233
2004 :	0.165	0.305
2005 :	0.256	0.100
2006 :	0.069	0.102
2007 :	0.093	0.092
2008 :	0.136	0.403
2009 :	0.089	0.458
2010 :	0.100	0.402
2011 :	0.022	0.274
2012 :	0.029	0.240
2013 :	0.323	0.355
2014 :	0.148	0.405
2015 :	0.100	0.369
2016 :	0.129	0.349
2017 :	0.400	0.503
2018 :	0.236	0.284
2019 :	0.350	0.258
2020 :	0.935	0.875
2021 :	0.626	0.567
2022 :	0.382	0.404
2023 :	0.324	0.324

Notes: p-values for joint F&CH-test for rolling 20-year windows ending in last year (inclusive 4th quarter). Using real-time data and currently available data.

5 Data

5.1 Core CPI

The monthly Core CPI inflation is the month-to-month percentage change of all items less food and energy of all urban consumers (CPI-U), U.S. city average, seasonally and not seasonally adjusted. Downloaded from Haver. The monthly change at an annual rate in percent is $100[(1+\pi/100)^{12}-1]$ where π is the month-to-month rate in percent.

5.2 Core PCE

The monthly Core PCE price index data are from the Personal Income and Outlays Tables of the NIPAs, the seasonally adjusted chained price indices for personal consumption expenditures, Table 2.8.4, downloaded from Haver. The Core price index is PCE excluding food and energy. Monthly inflation at an annual rate in percent is the log difference times 12×100.

The BEA started to publish NSA data only in 2018, as part of NIPA Table 8, cf Does BEA produce not seasonally adjusted estimates of GDP and GDI? But even now the NSA data are limited to quarterly main expenditure components. So, monthly NSA PCE indices are not available. The new NSA data and SA procedures are described in Preview of the 2018 Comprehensive Update of the National Income and Product Accounts with sections on *Improvements in seasonal adjustment* and *Introduction of not seasonally adjusted estimates*. The latter section states that BEA now provides *quarterly estimates of GDP, GDI, and their major components that are not seasonally adjusted (NSA)*, Table 8. Further information is in Chapter 4 of the Methodology Handbook. And for the SA of overall GDP How does BEA account for seasonality in GDP? notes that

> "Much of the data used by BEA to estimate detailed components of GDP are seasonally adjusted by the source data agencies. For example, BEA uses seasonally adjusted inventory and retail sales data from the U.S. Census Bureau and seasonally adjusted consumer price indexes from the U.S. Bureau of Labor Statistics. BEA does seasonally adjust some data itself, such as Treasury data used to measure federal government spending. **These seasonally adjusted detailed components are then aggregated to derive seasonally adjusted GDP." (My emphasis.)**

I assume that the same procedure applies more generally, that is, the BEA's SA procedure is bottom-up. The most detailed series are SA, and then aggregates are constructed from SA component series.

5.3 Real-time data

- Real-time data for monthly Core CPI inflation are calculated from real-time series of the monthly Core CPI index, file `pcpixMvMd.xlsx`. Monthly inflation at an annual rate is defined as the log difference times 100×12.

- Real-time data for quarterly Core PCE inflation are calculated from real-time series of the quarterly Core PCE index, file `pconxqvqd.xlsx`.

Both files were downloaded from the Real Time Data Set for Macroeconomists hosted by the Federal Reserve Bank of Philadelphia.

References

Canova, F. and B. E. Hansen (1995). Are seasonal patterns constant over time? a test for seasonal stability. *Journal of Business & Economic Statistics 13*(3), 237–252.

Cho, C.-K. and T. J. Vogelsang (2017). Fixed-b inference for testing structural change in a time series regression. *Econometrics 5*(1).

Cowan, B., S. Smith, and S. Thompson (2018, August). Seasonal adjustment in the national income and product accounts: Results from the 2018 comprehensive update. *Survey of Current Business 98*(8).

Kiefer, N. M. and T. J. Vogelsang (2005). A new asymptotic theory for heteroskedasticity-autocorrelation robust tests. *Econometric Theory 21*(6), 1130–1164.

Wright, J. H. (2018). Seasonal adjustment of NIPA data. Technical report, NBER WP 24895.