

# Trend Inflation, Firm-Specific Capital, and Sticky Prices

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Research on monetary policy, both at academic and monetary policy institutions, has increasingly been performed within an analytical framework that assumes limited nominal price adjustment, “sticky prices” for short. At the heart of much of this analysis is a so-called New Keynesian (NK) “expectational” Phillips curve that relates current inflation,  $\pi_t$ , to expected future inflation and the deviation of marginal cost from trend  $\hat{s}_t$ :

$$\pi_t = \beta E_t \pi_{t+1} + \xi \hat{s}_t, \quad (1)$$

with  $\beta, \xi > 0$ . Empirical estimates of the coefficient on the marginal cost term,  $\xi$ , in this NK Phillips curve tend to be positive but small in absolute value, e.g., Sbordone (2002) and Galí and Gertler (1999).<sup>1</sup> This represents a problem for the sticky-price framework since the coefficient  $\xi$  is directly related to the frequency with which nominal prices are assumed to be adjusted: the coefficient is smaller the less frequently prices are adjusted. Within standard sticky-price models, estimated values of  $\xi$  imply that prices are adjusted less than once per year. This macro estimate of price stickiness is implausibly high from the perspective of the micro estimates surveyed in Wolman (forthcoming).

It has been conjectured widely that nominal rigidities, such as sticky prices, have more persistent real effects if they interact with real rigidities. For example, the basic NK Phillips curve (1) has been derived for an environment with nominal frictions, but essentially no real rigidities: firms rent factors

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<sup>1</sup> Expression (1) is derived in Woodford (2003, ch. 3) for an economy with Calvo-type sticky prices. Woodford’s (2003) textbook presents a unified framework for thinking about monetary policy based on sticky-price models. For a critical review of this line of research, see Green (2005).

of production—capital and labor—in frictionless markets. Now, suppose that there is a real rigidity in addition to the sticky prices. In particular, assume that capital is specific to individual firms, and it is costly for these firms to adjust their capital stock. Introducing firm-specific capital adjustment costs into sticky-price models substantially complicates the analysis, yet Woodford (2005) manages to derive an almost closed-form solution to this problem. In particular, Woodford (2005) again derives an NK Phillips curve of the form (1), but now the marginal cost coefficient,  $\xi$ , depends not only on the extent of price stickiness, but also on the magnitude of capital adjustment costs: the coefficient is smaller the less frequently prices are adjusted and the more costly it is to adjust capital. Thus low estimated values of  $\xi$  do not necessarily imply a high degree of price stickiness.

Woodford's (2005) clean analytical solution of the modified NK Phillips curve does come with a cost. His approach is based on the linear approximation of an economy with Calvo-type nominal price adjustment around an equilibrium with zero average inflation. The assumption of zero average inflation makes the theoretical analysis of the firm aggregation problem possible, yet it is not empirically plausible. Even though in recent years inflation has been remarkably stable in many industrialized countries, average inflation has been positive. Furthermore, most estimates of the NK Phillips curve use data from periods of moderate inflation. Thus, it is important to know whether the behavior of these models is sensitive to the steady-state inflation rate.<sup>2</sup>

In this article we evaluate the relative impact of positive average inflation versus zero inflation in an economy with nominal rigidities and firm-specific capital adjustment costs. Unlike Woodford (2005), we model nominal rigidities as Taylor-type staggered price adjustment, and not as Calvo-type probabilistic price adjustment. This approach is necessary since at this time there are no aggregation results for our economic environment with Calvo-type pricing and nonzero average inflation. We show that for small values of positive average inflation, the Taylor principle, which states that a central bank should increase the nominal interest rate more than one-for-one in response to a deviation of inflation from its target, is no longer sufficient to guarantee that monetary policy does not become a source of unnecessary fluctuations in our economy.

The fundamental difficulty with incorporating firm-specific capital into a model with sticky prices is that firm-specific capital can amplify the heterogeneity associated with price stickiness. With Calvo price setting, firms face a constant exogenous probability of being able to readjust their price. If there

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<sup>2</sup> Furthermore, even though overall inflation has been low and stable, trends have remained in disaggregated measures of prices—for example, services prices have a positive trend and durable goods prices have a negative trend. This means that in a multi-sector model with zero inflation, the steady state would involve trends in individual nominal prices and thus a nondegenerate distribution of prices across sticky-price firms (Wolman 2004).

are no state variables specific to the firm (other than price), then all firms that adjust in a given period choose the same price. In that case, even though the true distribution of prices is infinite, it is possible to summarize the relevant distribution with just a small number of state variables.<sup>3</sup> If instead capital is firm specific, firms that adjust in the same period generally do not have the same capital stock. Their marginal cost is not the same, and in general they will not choose the same price. Thus, combining Calvo pricing and firm-specific capital appears to lead to an intractable model.

The model is intractable in its exact form, but Sveen and Weinke (2004) and Woodford (2005) have shown how to derive a tractable linear approximation to the model, under the assumption that the average inflation rate is zero. The key to these derivations is the fact that in the zero-inflation steady state there is no heterogeneity: all firms charge the same price.

Given the tractability problem, there is little hope of being able to learn how the Calvo model with firm-specific capital behaves away from a zero-inflation steady state. Fortunately, there is another class of sticky-price models that remains tractable when combined with firm-specific capital. The staggered pricing framework associated with Taylor (1980) assumes that there are  $J$  different types of firms; each period a fraction  $1/J$  of firms adjusts their prices, and their prices remain fixed for  $J$  periods. Firm-specific capital presents no problems in the Taylor model, because it remains the case that all firms that adjust in a given period enter with the same capital stock and thus will choose the same price.

We solve the linear approximation to the Taylor model numerically and ask whether the model's dynamics are sensitive to the steady-state inflation rate around which we linearize.<sup>4</sup> We find that a small but positive inflation rate can have a big impact on the set of parameters for monetary policy rules and investment adjustment costs for which a rational expectations (RE) equilibrium is unique.<sup>5</sup> If the equilibrium is not unique, that is, there is equilibrium indeterminacy, then possible equilibrium outcomes can depend on shocks that do not constrain the resource feasible allocations in an economy. In these equilibria self-fulfilling expectations that coordinate on such nonfundamental shocks, known as "sunspots," introduce unnecessary fluctuations into the economy.

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<sup>3</sup> We say the true distribution is infinite because a positive fraction of firms charges a price set arbitrarily many periods in the past.

<sup>4</sup> Others have worked with the Taylor model with firm-specific capital; see, for example, Coenen and Levin (2004) and de Walque, Smets, and Wouters (2004). They have not studied the role of steady-state inflation.

<sup>5</sup> Since we are studying linear approximations of equilibria, all of our statements have to be understood as applying to local properties of the equilibria for small deviations from the steady state. Wolman and Couper (2003) discuss the potential pitfalls of this type of analysis, especially as it relates to statements about the uniqueness of equilibrium.

In standard sticky-price models, monetary policy rules that set the nominal interest rate in response to deviations of inflation from its target value achieve a unique RE equilibrium, if they follow the Taylor principle. The principle states that the nominal interest rates increase more than one-for-one with an increase of the inflation rate. This policy response does not have to be very big, as long as it is greater than one. We show that in the sticky-price model with firm-specific capital, positive steady-state inflation generally increases the region of the parameter space for which there is indeterminacy of equilibrium. In other words, for the same magnitudes of price-stickiness and capital-adjustment costs, monetary policy has to be much more responsive to deviations of inflation from its target in order to maintain a unique RE equilibrium outcome. These results suggest that it may be misleading to interpret history and make policy recommendations based on findings from the zero steady-state inflation case. Our results complement those in Sveen and Weinke (2005), who show that moving from a rental market to firm-specific capital leads to a larger region of the parameter space for which there is indeterminacy of equilibrium when steady-state inflation is zero.

In Section 1 we describe the economy with firm-specific capital adjustment cost and the two types of sticky prices: Calvo-type and Taylor-type nominal price setting. In Section 2 we outline how Woodford (2005) solves the aggregation problem for Calvo-type pricing and derives the modified NK Phillips curve. In Section 3 we characterize the economy with Taylor-type pricing, and in Section 4 we study the impact of capital adjustment costs and nonzero average inflation on the economy with Taylor-type pricing.

## 1. STICKY-PRICE MODELS WITH FIRM-SPECIFIC CAPITAL

This section presents the common features of Calvo and Taylor sticky-price models. There is an infinitely lived representative household and a continuum of differentiated firms. The firms act as monopolistic competitors in their differentiated output markets, but they are competitive in their differentiated labor markets. The differentiated output goods of the firms are used to produce an aggregate output good in a competitive market. The aggregate output good can be used for consumption or investment. Firms use investment goods to augment their firm-specific capital stocks, subject to capital adjustment costs. Firms set the nominal price of their differentiated output good, and only infrequently do they have the opportunity to adjust their nominal price.

### The Representative Household

The household values consumption,  $c_t$ , and experiences disutility from the supply of differentiated labor to a continuum of markets,  $h_t(j)$ . The expected

present value of utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \gamma \int_0^1 \frac{h_t(j)^{\nu+1}}{\nu+1} dj \right\}, \quad (2)$$

with discount factor,  $\beta$ . Period utility is an increasing (decreasing) concave (convex) function of consumption (work time),  $\sigma, \nu, \gamma > 0$ . The representative household owns shares in the continuum of firms and holds nominal bonds. The household's budget constraint is

$$\begin{aligned} P_t c_t + \int_0^1 Q_t(j) a_{t+1}(j) dj + B_{t+1} &= \int_0^1 W_t(j) n_t(j) dj \\ &+ \int_0^1 [Q_t(j) + D_t(j)] a_t(j) dj + (1 + i_t) B_t, \end{aligned} \quad (3)$$

where  $P_t$  is the nominal price of the aggregate output good,  $Q_t(j)$  is the nominal price of a share in firm  $j$ ,  $W_t(j)$  is the nominal wage paid by firm  $j$ ,  $D_t(j)$  is the nominal dividend paid by firm  $j$ ,  $i_t$  is the nominal interest rate on nominal bond holdings  $B_t$ , and  $a_t(j)$  is the household's firm-share holdings.

Optimal choice of work effort implies the following firm-specific labor supply functions

$$w_t(j) = \gamma h_t(j)^\nu / \lambda_t, \quad (4)$$

where  $w_t(j) = W_t(j) / P_t$  is the real wage paid by firm  $j$ , and  $\lambda_t$  is marginal utility of consumption

$$\lambda_t = c_t^{-\sigma}. \quad (5)$$

Optimal asset and bond holdings imply the following Euler equations for bonds and firm shares

$$1 = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + i_t}{P_{t+1}/P_t} \right] \text{ and} \quad (6)$$

$$1 = E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{[Q_{t+1}(j) + D_{t+1}(j)] / Q_t(j)}{P_{t+1}/P_t} \right]. \quad (7)$$

The representative household chooses consumption such that the household is indifferent between consuming slightly more, with a corresponding reduction in asset holdings, and consuming slightly less, with a corresponding increase in asset holdings. The Euler equations embody this indifference. In an equilibrium, the representative household owns all firms,  $a_t(j) = 1$ .

### Aggregate Output

The aggregate output,  $y_t$ , is produced from the continuum of differentiated inputs,  $y_t(j)$ , using a constant-elasticity-of-substitution production function

$$y_t = \left[ \int_0^1 y_t(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}, \quad (8)$$

where  $\theta \geq 1$  denotes the elasticity of substitution between goods. This is the Dixit-Stiglitz (1977) formulation used by Blanchard and Kiyotaki (1987). Production is competitive and given nominal prices,  $P_t(j)$ , for the differentiated inputs, cost minimization implies the following nominal price index/marginal cost for the aggregate output

$$P_t = \left[ \int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \quad (9)$$

Given aggregate output, the demand for a differentiated good is a function of its relative price,  $p_t(j) \equiv P_t(j)/P_t$ ,

$$y_t(j) = p_t(j)^{-\theta} y_t. \quad (10)$$

Aggregate output can be used for consumption or for the accumulation of firm-specific capital by the producers of differentiated goods,  $x_t(j)$ . Market clearing for goods implies that aggregate output equals the sum of consumption and aggregate investment

$$y_t = c_t + \int_0^1 x_t(j) dj. \quad (11)$$

### Firms

The differentiated goods are produced by a continuum of monopolistically competitive firms, and these are the same firms to which the household supplies labor. The differentiated goods are produced using the inputs capital and labor, both of which are specific to each firm. The differentiated firms can adjust the nominal prices they set for their product only infrequently.

#### *Production*

Production is constant-returns-to-scale; in particular, we assume that the production function is Cobb-Douglas:

$$y_t(j) = k_t(j)^\alpha [A_t h_t(j)]^{1-\alpha}; \quad (12)$$

$y_t(j)$  is firm  $j$ 's output in period  $t$ , and  $k_t(j)$  and  $h_t(j)$  are, respectively, the capital input and labor input used by firm  $j$  in period  $t$ . There is an aggregate productivity disturbance given by  $A_t$ . At the beginning of period  $t$ , firm  $j$ 's capital input is predetermined as a result of the investment decision firm  $j$  made

in period  $t - 1$ . Furthermore, there are convex costs of changing the capital stock, which we will specify further below. Labor is hired in competitive markets, but because households receive distinct disutility from the labor they provide to each firm, the wage can differ across firms.<sup>6</sup>

In order to change its capital stock from  $k_t$  in period  $t$  to  $k_{t+1}$  in period  $t + 1$ , a firm needs  $x_t$  units of the aggregate output good

$$x_t(j) = k_t(j) G[k_{t+1}(j)/k_t(j)]. \quad (13)$$

The firm incurs capital adjustment costs determined by the increasing and convex function,  $G(k_{t+1}/k_t)$ . As in Woodford (2005),  $G(1) = \delta$ ,  $G'(1) = 1$  and  $G''(1) = \epsilon_\psi$ , where  $\epsilon_\psi > 0$  is a parameter. If the firm exactly replaces depreciated capital, then the marginal investment cost is one, but if the firm increases its capital stock, then the marginal cost of each additional unit of capital is greater than one and increasing with the rate at which the capital stock increases.

### Prices

Firms in the model face limited opportunities for price adjustment. In particular, we assume that any firm faces an exogenous probability of adjusting its price in period  $t$  and that the probability may depend on when the firm last adjusted its price. The key notation describing limited price adjustment will be a vector  $\Phi$  (possibly with a countably infinite number of elements); the  $s^{\text{th}}$  element of  $\Phi$ , called  $\phi_s$ , is the probability that a firm adjusts its price in period  $t$ , conditional on its previous adjustment having occurred in period  $t - s$ .

There is a time invariant distribution of firms according to when they last adjusted their price, since the price-adjustment probabilities do not vary with time. Let  $\omega_s$  denote the fraction of firms in period  $t$ , charging prices set in periods  $t - s$ , with the corresponding vector,  $\Omega$ . Given the price-adjustment probabilities, the time invariant distribution satisfies

$$\begin{aligned} \omega_s &= (1 - \phi_s) \omega_{s-1}, \text{ for } s = 1, 2, \dots, \text{ and} \\ \omega_0 &= 1 - \sum_{s=1}^{J-1} \omega_s. \end{aligned} \quad (14)$$

The most common pricing specifications in the literature are those first described by Taylor (1980) and Calvo (1983). Taylor's specification is one of uniformly staggered price setting: every firm sets its price for  $J$  periods, and at any point in time a fraction  $1/J$  of firms charge a price set  $s$  periods ago. The  $J$ -element vector of adjustment probabilities for the Taylor model is  $\Phi = [0, \dots, 0, 1]$ , and the  $J$ -element vector of fractions of firms is

<sup>6</sup> Labor market clearing is implicitly imposed by not differentiating between the labor supplied to the  $j^{\text{th}}$  type of firm and the labor demanded by the  $j^{\text{th}}$  type of firm.

$\Omega = [1/J, 1/J, \dots, 1/J]$ . In contrast, Calvo's specification involves uncertainty about when firms can adjust their price. No matter when a firm last adjusted its price, it faces a probability  $\phi$  of adjusting. Thus, the infinite vector of adjustment probabilities is  $\Phi = [\phi, \phi, \dots]$ , and the infinite vector of fractions of firms is  $\omega_s = \phi (1 - \phi)^s$ ,  $s = 0, 1, \dots$

### **Firm Value**

We assume that a firm pays out each period's profits as dividends to its shareholders:

$$d_t(j) = p_t(j) y_t(j) - w_t(j) h_t(j) - x_t(j). \quad (15)$$

Conditional on the firm's relative price,  $p_t(j)$ , sales,  $y_t(j)$ , are determined by the demand curve (10). The firm's demand for labor is

$$h(j) = H[y(j), k(j), A] = \left[ \frac{y(j)}{k(j)^\alpha} \right]^{1/(1-\alpha)} A^{-1}. \quad (16)$$

The rationale behind solving for labor input in (16) is that in period  $t$  the firm's capital stock is predetermined, and thus the labor input it must employ is determined by its technology, given the level of demand,  $y_t(j)$ . Conditional on the available capital stock, the marginal (labor) cost of output is then

$$s_t(j) = \frac{1}{1-\alpha} \frac{w_t(j) y_t(j)}{h_t(j)}. \quad (17)$$

Investment is determined by the capital stock the firm operates at the beginning of the period and the capital stock the firm plans to operate in the next period, equation (13). With some abuse of notation we can rewrite the real dividends of a firm as a function of its idiosyncratic state and control variables: the relative price and the beginning-of-period and end-of-period capital stocks,

$$d_t(j) = d_t[p_t(j), k_t(j), k_{t+1}(j)]. \quad (18)$$

The dependence on the aggregate state of the economy (aggregate demand, productivity, wages) is subsumed in the time subscript  $t$  for the function  $d$ .

The firms maximize the discounted expected present value of future dividends. The relevant discount factor is the representative household's intertemporal marginal rate of substitution, since the firms are owned by the household,

$$\max E_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{\lambda_{t+\tau}}{\lambda_t} d_{t+\tau}(j). \quad (19)$$

Let  $v_t(p_{-1}, k, j)$  denote the value of a firm with relative price,  $p_{-1}$ , in the last period and beginning of period capital stock  $k$ . Let  $j$  denote when the firm last adjusted its nominal price. If  $j = 0$ , the firm can adjust its nominal price in the current period, that is,  $p_{-1}$  does not affect the firm's value and we



write  $v_t(k, 0)$ . We can write the value of a firm as a function of its own state variables recursively,

$$v_t(k_t, 0) = \max_{p_t^*, k_{t+1}} \left\{ d_t(p_t^*, k_t, k_{t+1}) + E \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \{ \phi_1 v_{t+1}(k_{t+1}, 0) + (1 - \phi_1) v_{t+1}(p_t^*, k_{t+1}, 1) \} \right] \right\}, \quad (20)$$

$$v_t(p_{t-1}, k_t, j) = \max_{k_{t+1}} \left\{ d_t(p_t, k_t, k_{t+1}) + E \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \{ \phi_{j+1} v_{t+1}(k_{t+1}, 0) + (1 - \phi_{j+1}) v_{t+1}(p_t, k_{t+1}, j + 1) \} \right] \right\}, \quad (21)$$

$$\text{and } p_t = p_{t-1} \frac{P_{t-1}}{P_t}$$

Note that for Calvo pricing,  $\phi_j = \phi$ , and therefore  $v_t(p_{-1}, k, 1) = v_t(p_{-1}, k, j)$  for all  $j \geq 1$ . On the other hand, for Taylor pricing the firm value functions are only defined for  $j \leq J - 1$ , since  $\phi_J = 1$ .

### Government Policy

We assume that there is neither taxation nor government spending. Monetary policy chooses a desired steady-state level for the inflation rate,  $\pi^*$ . Given the steady-state real interest rate,  $1/\beta$ , the steady-state nominal interest rate,  $i^*$ , consistent with the inflation rate,  $\pi^*$ , is

$$1 + i^* = \frac{1 + \pi^*}{\beta}. \quad (22)$$

Monetary policy is assumed to set the period nominal interest rate in response to deviations of the inflation rate and output from their respective steady-state values,

$$i_t = i^* + f_\pi [P_t/P_{t-1} - (1 + \pi^*)] + f_y \left[ \frac{y_t - y^*}{y^*} \right]. \quad (23)$$

## 2. THE CALVO MODEL

We now outline how the equilibrium of the economy with Calvo pricing can be characterized for a log-linear approximation around a steady state with zero inflation. In particular, we show that despite the fact that firms differ according to their relative prices and their capital stocks, calculating simple averages over all these firms yields a consistent aggregation. We do not provide a complete characterization of the equilibrium; for this we refer the reader to Woodford (2005). Although our results below on equilibrium indeterminacy are for the Taylor model, we present the equilibrium characterization for the Calvo model

because it helps to explain the appeal of firm-specific capital. It is only in the zero-inflation Calvo model that one can solve for a simple NK Phillips curve involving aggregate marginal cost and see how the coefficient on marginal cost depends on investment adjustment costs as well as price stickiness.

The crucial element of the procedure is that the approximation proceeds around a deterministic steady state where all firms are identical, so that the log-linearized first-order conditions are the same for all firms. This feature makes it possible to derive a first-order aggregation over firms that may temporarily deviate from the deterministic steady state, and may therefore be characterized by firm-specific state variables,  $k_t(j)$  and  $P_t(j)$ .

Since firms differ only because they may or may not have the chance to adjust their prices, there are only two possibilities for firms to be the same in the steady state despite the fact that they do not all adjust their prices at the same time. First, there is zero steady-state inflation. In this case there is no need for firms to adjust their prices and they will all be the same anyway. Second, there is indexation: if firms cannot adjust their price optimally to their current state, their price is nevertheless adjusted according to the average inflation rate. Thus the firm's relative price also does not change. In the following we study the first case, zero steady-state inflation.

To summarize, we study the log-linear approximation of an economy with a deterministic steady state where all firms are identical. That is, we have  $p_t^{ss}(j) = 1$  and  $k_t^{ss}(j) = k^*$ .

### Optimal Capital Accumulation

Taking the firm's price decision as given for the time being, optimal choices of  $k_{t+1}(j)$  and  $x_t(j)$  maximize the expectation of (19) subject to the firm's product demand function (10), capital adjustment costs (13), and demand for labor (16).

The first-order conditions for  $k_{t+1}$  imply the following Euler equation:

$$\begin{aligned} & G' \left( \frac{k_{t+1}(j)}{k_t(j)} \right) \\ &= E_t \left[ \beta \frac{\lambda_{t+1}}{\lambda_t} \left\{ G \left( \frac{k_{t+2}(j)}{k_{t+1}(j)} \right) \cdot \left( \frac{G' [k_{t+2}(j) / k_{t+1}(j)]}{G [k_{t+2}(j) / k_{t+1}(j)]} \cdot \frac{k_{t+2}(j)}{k_{t+1}(j)} - 1 \right) \right. \right. \\ & \quad \left. \left. + u_{t+1}(j) \right\} \right], \end{aligned} \tag{24}$$

where  $u_{t+1}(j)$  denotes the value of having an additional unit of capital in period  $t + 1$ . This value,  $u$ , is the marginal labor cost reduction from the

additional capital:

$$\begin{aligned} u_{t+1}(j) &= -w_{t+1}(j) \frac{\partial H[y_{t+1}(j), k_{t+1}(j), A_{t+1}]}{\partial k_{t+1}(j)} \\ &= \frac{\alpha}{1-\alpha} w_{t+1}(j) h_{t+1}(j) / k_{t+1}(j). \end{aligned} \quad (25)$$

The Euler equation is somewhat complicated, but it embodies the fact that a marginal increase in next period's capital stock has three effects. It subtracts from resources available for current consumption; it adds to resources available for future consumption; and it reduces future labor costs.

We now derive the log-linear approximation of the firm's Euler equation for capital (24). Let  $\hat{x}$  denote the percentage deviation of a variable from its steady-state value  $x^*$ ,  $\hat{x} = dx/x^*$ . Because  $k_{t+1}^{ss}(j)/k_t^{ss}(j) = 1$ , the log-linear approximation of the Euler equation is

$$\begin{aligned} \frac{G''(1)}{G'(1)} [\hat{k}_{t+1}(j) - \hat{k}_t(j)] &= E_t \left[ \beta \frac{G''(1)}{G'(1)} [\hat{k}_{t+2}(j) - \hat{k}_{t+1}(j)] \right. \\ &\quad \left. + [1 - \beta(1 - \delta)] \hat{u}_{t+1}(j) + \hat{\lambda}_{t+1} - \hat{\lambda}_t \right]. \end{aligned} \quad (26)$$

Note that  $G''(1)/G'(1) = \epsilon_\psi$ . The log-linear approximation of the marginal value of capital (24) is

$$\hat{u}_{t+1}(j) = \hat{w}_{t+1}(j) + \hat{h}_{t+1}(j) - \hat{k}_{t+1}(j). \quad (27)$$

After substituting for firm-specific labor supply using (5), this equation can be written as

$$\hat{u}_{t+1}(j) = \left[ v \hat{h}_{t+1}(j) - \hat{\lambda}_{t+1} \right] + \hat{h}_{t+1}(j) - \hat{k}_{t+1}(j). \quad (28)$$

Next, substituting for the equilibrium employment from (16) and then substituting for firm  $j$ 's output using the demand function (10), we get the marginal value of a unit of firm-specific capital in terms of the firm-specific variables (relative price and capital stock) and the aggregate variables (aggregate demand, marginal utility, and technology):

$$\begin{aligned} \hat{u}_{t+1}(j) &= -\theta \frac{v+1}{1-\alpha} \hat{p}_{t+1}(j) - \left[ \frac{(v+1)\alpha}{1-\alpha} + 1 \right] \hat{k}_{t+1}(j) \\ &\quad + \frac{v+1}{1-\alpha} \hat{y}_{t+1} - \hat{\lambda}_t - (v+1) \hat{A}_{t+1}. \end{aligned} \quad (29)$$

Notice that the Euler-equation approximations (26) and (29) are the same for all firms, independent of their idiosyncratic state. We can now average/aggregate over these approximate first-order conditions of all firms. For the following, let

$$\hat{k}_t \equiv \int_0^1 \hat{k}_t(j) di \quad (30)$$

be the deviation of the aggregate capital stock from its steady-state value, and similarly for all other variables. Aggregating over the first-order conditions (26) and (29), we have

$$\begin{aligned} \epsilon_\psi \left( \hat{k}_{t+1} - \hat{k}_t \right) &= E_t \left[ \hat{\lambda}_{t+1} - \hat{\lambda}_t + \beta \epsilon_\psi \left( \hat{k}_{t+2} - \hat{k}_{t+1} \right) \right. \\ &\quad \left. + \{1 - \beta (1 - \delta)\} \hat{u}_{t+1} \right]; \end{aligned} \quad (31)$$

$$\begin{aligned} \hat{u}_{t+1} &= \frac{\nu + 1}{1 - \alpha} \hat{y}_{t+1} - \hat{\lambda}_{t+1} - \left[ \frac{(\nu + 1)\alpha}{1 - \alpha} + 1 \right] \hat{k}_{t+1} \\ &\quad - (\nu + 1) \hat{A}_{t+1}. \end{aligned} \quad (32)$$

For the aggregate marginal value of capital we have used the fact that (9) implies

$$\int_0^1 \hat{p}_t(j) dj = 0. \quad (33)$$

Now define a firm's capital stock deviation from the aggregate deviation from the steady state as

$$\tilde{k}_t(j) = \hat{k}_t(j) - \hat{k}_t \quad (34)$$

and subtract the aggregate conditions (31) and (32) from the firm-specific conditions (26) and (29) to yield

$$\begin{aligned} \epsilon_\psi \left\{ \tilde{k}_{t+1}(j) - \tilde{k}_t(j) \right\} &= E_t \left[ \beta \epsilon_\psi \left\{ \tilde{k}_{t+2}(j) - \tilde{k}_{t+1}(j) \right\} \right. \\ &\quad \left. + \{1 - \beta (1 - \delta)\} \tilde{u}_{t+1}(j) \right], \end{aligned} \quad (35)$$

$$\begin{aligned} \tilde{u}_{t+1}(j) &= -\frac{\nu + 1}{1 - \alpha} \theta \hat{p}_{t+1}(j) \\ &\quad - \left\{ \frac{(\nu + 1)\alpha}{1 - \alpha} + 1 \right\} \tilde{k}_{t+1}(j). \end{aligned} \quad (36)$$

Note that (35) and (36) define an autonomous system for the firm-specific relative capital stock and relative price that is independent of aggregate variables. In order to complete this system, we need the expression for the unconditional expectation of the firm's relative price in the next period. There are two possibilities for next period's relative price. First, with probability  $1 - \phi$ , the firm will be unable to adjust its nominal price, and its relative price declines with the aggregate inflation rate  $\pi$ . Second, with probability  $\phi$ , the firm can adjust its nominal price and the optimal relative price choice is  $\hat{p}^*$ :

$$E_t \hat{p}_{t+1}(j) = (1 - \phi) \left[ \hat{p}_t(j) - E_t \pi_{t+1} \right] + \phi E_t \hat{p}_{t+1}^*(j). \quad (37)$$

The analysis so far suggests that we can solve for the evolution of the firm's relative state variables independently of the evolution of aggregate state variables, but it also implies that optimal capital accumulation and optimal price setting will interact.

### The Interaction of Price Setting and Capital Accumulation

We first show how aggregate inflation is related to the average price chosen by all the firms that can adjust prices. Once we conjecture that a particular price-adjusting firm's deviation from this average optimal price depends only on its relative capital stock, we can show how to solve for the evolution of the firm's relative capital stock. Conditional on the law of motion for the firm's optimal relative capital stock, one can then solve the firm's optimal price-setting problem. For an equilibrium, the conjecture on the optimal price-setting rule in the first step has to be consistent with the solution of the price-setting problem in the second step. This second step involves quite a bit of algebra, and we refer the reader to Woodford (2005) for the solution. We do state the Phillips curve equation that follows from these steps. The form of the Phillips curve illustrates the appeal of firm-specific capital.

#### Aggregate Inflation

In the Calvo setup, aggregate inflation is determined as a weighted average of the current distribution of relative prices and the optimal relative prices set by price-adjusting firms. At the beginning of period  $t + 1$ , a fraction  $1 - \phi$  of all firms keeps their price and a fraction  $\phi$  adjusts their price conditional on their state. For both groups we can use the unconditional distribution of all firms in the economy. Thus, the deviation of the aggregate price level from the steady state is

$$\hat{P}_{t+1} = (1 - \phi) \int_0^1 \hat{P}_t(j) dj + \phi \int_0^1 \hat{P}_t^*(j) dj = (1 - \phi) \hat{P}_t + \phi \hat{P}_{t+1}^*. \quad (38)$$

Subtract  $\hat{P}_t$  from both sides and the aggregate inflation rate is

$$\pi_{t+1} = \hat{P}_{t+1} - \hat{P}_t = \phi \left( \hat{P}_{t+1}^* - \hat{P}_t \right). \quad (39)$$

Adding and subtracting  $\hat{P}_{t+1}$  on the right-hand side and using the definition of the inflation rate, we get the inflation rate proportional to the average optimal relative price

$$(1 - \phi) \pi_{t+1} = \phi \left( \hat{P}_{t+1}^* - \hat{P}_{t+1} \right) = \phi \hat{p}_{t+1}^*. \quad (40)$$

Using expression (40) for the inflation rate in the definition of next period's unconditional expected relative price (37) we get

$$\begin{aligned} E_t \hat{p}_{t+1}(j) &= (1 - \phi) \left( \hat{p}_t(j) - E_t \left[ \frac{\phi}{1 - \phi} \hat{p}_{t+1}^* \right] \right) + \phi E_t^* \hat{p}_{t+1}(j) \\ &= (1 - \phi) \hat{p}_t(j) + \phi E_t \left[ \hat{p}_{t+1}^*(j) - \hat{p}_{t+1} \right]. \end{aligned} \quad (41)$$

Now assume that the deviation of a firm's optimal relative price from the average optimal relative price is a function of the firm's relative state only:

$$\hat{p}_t^*(j) = \hat{p}_t^* - \mu \tilde{k}_t(j). \quad (42)$$

Then equations (35), (36), (41), and (42) define an autonomous system for the firm-specific relative capital stock,  $\tilde{k}(j)$ , and relative price,  $\hat{p}(j)$ , that is independent of aggregate variables. We are interested in a recursive solution to this system, that is, a solution such that the firm's choice for next period's relative capital stock,  $\tilde{k}_{t+1}(j)$ , is a function of its own relative state only,  $[\tilde{k}_t(j), \hat{p}_t(j)]$ :

$$\tilde{k}_{t+1}(j) = \Lambda \tilde{k}_t(j) - \tau \hat{p}_t(j). \quad (43)$$

### *Optimal Price Setting*

Woodford (2005) solves the optimal price-setting problem conditional on the optimal capital accumulation rule (43). In particular, the optimal price-setting rule is shown to be of the form assumed in equation (42): the deviation of a particular firm's optimal relative price from the average optimal relative price,  $\hat{p}_t^*(i) - \hat{p}_t^*$ , is a function of the firm's relative state,  $\tilde{k}_t(i)$ . Woodford (2005) shows how one can obtain the coefficients  $\Lambda$ ,  $\tau$ , and  $\mu$  through the method of undetermined coefficients.

The solution of the optimal pricing problem yields an expression for the average optimal price as a function of the average marginal labor cost of production,  $\hat{s}_t$ , and expected future optimal prices and inflation:

$$\hat{p}_t^* = \frac{1 - (1 - \phi)\beta}{\Gamma} \hat{s}_t + (1 - \phi)\beta E_t [\pi_{t+1} + \hat{p}_{t+1}^*], \quad (44)$$

where  $\Gamma$  is a coefficient to be determined by the solution procedure. In particular,  $\Gamma$  will depend on the price-adjustment probability  $\phi$  and the degree of capital adjustment costs,  $\epsilon_\psi$ . Average marginal cost is by definition

$$\begin{aligned} \hat{s}_t &\equiv \int_0^1 [\hat{w}_t(j) + \hat{h}_t(j) - \hat{y}_t(j)] dj \\ &= \left( \frac{\nu + 1}{1 - \alpha} - 1 \right) \hat{y}_t - \hat{\lambda}_t - (\nu + 1) \left[ \frac{\alpha}{1 - \alpha} \hat{k}_t + \hat{A}_t \right]. \end{aligned} \quad (45)$$

We can now use again the expression for aggregate inflation in the Calvo model in (40) and derive the "standard" New Keynesian Phillips curve

$$\pi_t = \frac{[1 - (1 - \phi)\beta]\phi}{(1 - \phi)\Gamma} \hat{s}_t + \beta E_t [\pi_{t+1}]. \quad (46)$$

For a simple Calvo model with no firm-specific capital,  $\Gamma = 1$ . Thus the modified Calvo model with firm-specific capital adjustment costs generates almost the same NK Phillips curve as the basic Calvo model, except for  $\Gamma$ .

In particular, higher capital adjustment costs increase  $\Gamma$  and thereby reduce the coefficient on the marginal cost term. Woodford (2005) and Eichenbaum and Fisher (2004) thus argue that a low estimated coefficient on marginal cost does not necessarily imply that the price-adjustment probability is very low; it can also mean that the capital adjustment costs are very high.

### 3. THE TAYLOR MODEL

In the Taylor model, price adjustment occurs every  $J$  periods for an individual firm, and in any given period by a fraction  $1/J$  of firms. Because there is no uncertainty regarding when a firm will adjust its price, the state space does not explode as it does in the Calvo model. Therefore, the Taylor model with firm-specific capital can be approximated easily around a steady state with nonzero inflation. Here we present the exact equations of the model. We then linearize them and compute the model's local dynamics.

#### Pricing

An individual firm that can adjust its price in period  $t$  chooses a sequence of nominal prices,  $\{P_{t+Js}^*(j)\}$ , every  $J$  periods, and a sequence of capital stocks  $\{k_{t+1}^*(j)\}$  every period, that maximizes the objective function

$$\max E_t \sum_{s=0}^{\infty} \beta^{Js} \sum_{\tau=0}^{J-1} \beta^{\tau} \frac{\lambda_{t+Js+\tau}}{\lambda_t} \times \quad (47)$$

$$\left\{ \left[ \frac{P_{t+Js}^*(j)}{P_{t+Js+\tau}} \right]^{1-\theta} y_{t+Js+\tau} - w_{t+Js+\tau}(j) h_{t+Js+\tau}(j) - x_{t+Js+\tau}(j) \right\},$$

subject to the demand for the firm's goods (10) and the firm's demand for labor (16). Note that in contrast to the Calvo model, the expectation operator in (47) is the unconditional expectation operator—there is no uncertainty in the price adjustment process. The first-order conditions for optimal price setting are

$$E_t \sum_{\tau=0}^{J-1} \beta^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} (1-\theta) \frac{1}{P_{t+\tau}} \left( \frac{P_t^*(j)}{P_{t+\tau}} \right)^{-\theta} y_{t+\tau}$$

$$+ \theta E_t \sum_{\tau=0}^{J-1} \beta^{\tau} s_{t+\tau}(j) \frac{\lambda_{t+\tau}}{\lambda_t} \frac{1}{P_{t+\tau}} \left( \frac{P_t^*(j)}{P_{t+\tau}} \right)^{-\theta-1} y_{t+\tau} = 0, \quad (48)$$

where  $s_t(j)$  is the firm's marginal (labor) cost of production, (17). The first-order conditions for optimal capital accumulation are the same as in the Calvo model, equations (25) and (26).

To simplify (48) we will solve for the optimal price  $P_t^*(j)$ , at the same time dividing both sides of the equation by  $P_t$  :

$$\frac{P_t^*(j)}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{\tau=0}^{J-1} \beta^\tau s_{t+\tau}(j) \frac{\lambda_{t+\tau}}{\lambda_t} \left( \frac{P_t}{P_{t+\tau}} \frac{1}{P_t} \right)^{-\theta} y_{t+\tau}}{E_t \sum_{\tau=0}^{J-1} \beta^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}} \left( \frac{P_t}{P_{t+\tau}} \frac{1}{P_t} \right)^{-\theta} y_{t+\tau}}. \quad (49)$$

Next, note that  $P_t^\theta$  cancels from the numerator and denominator:

$$\frac{P_t^*(j)}{P_t} = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{\tau=0}^{J-1} \beta^\tau s_{t+\tau}(j) \frac{\lambda_{t+\tau}}{\lambda_t} \left( \frac{P_{t+\tau}}{P_t} \right)^\theta y_{t+\tau}}{E_t \sum_{\tau=0}^{J-1} \beta^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left( \frac{P_{t+\tau}}{P_t} \right)^{\theta-1} y_{t+\tau}}. \quad (50)$$

Until now we have carried around the firm's index  $j$ , which lies in the interval  $[0, 1]$ . However with Taylor pricing, it is only necessary to keep track of  $J$  different types of firms—any firms that set their price in the same period behave identically. Of course, this is not the case in the Calvo model.<sup>7</sup> Henceforth the index  $j$  denotes the finite types  $J$ . For example, the marginal cost for a firm that set its price in period  $t - j$  will be  $s_{j,t}$ ; the price in period  $t$  charged by a firm that last set its price in period  $t - j$  will be  $P_{j,t}$ . Thus, instead of  $P_t^*(j)$  we will write  $P_{0,t}$ .

$$\frac{P_{0,t}}{P_t} = \frac{\theta}{\theta - 1} \cdot \frac{E_t \sum_{j=0}^{J-1} \beta^j s_{j,t+j} \lambda_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^\theta y_{t+j}}{E_t \sum_{j=0}^{J-1} \beta^j \lambda_{t+j} \left( \frac{P_{t+j}}{P_t} \right)^{\theta-1} y_{t+j}}. \quad (51)$$

Imposing the fact that there are only  $J$  prices charged, the price index can be written as

$$P_t = \left\{ \frac{1}{J} \sum_{j=0}^{J-1} P_{0,t-j}^{1-\theta} \right\}^{\frac{1}{1-\theta}}, \quad (52)$$

and the demand equations are

$$y_{j,t} = p_{j,t}^{-\theta} y_t, \quad j = 0, 1, \dots, J - 1. \quad (53)$$

Also, from the household side we have the labor supply equations

$$\frac{\gamma h_{j,t}^\nu}{\lambda_t} = w_{j,t}, \quad j = 0, 1, \dots, J - 1. \quad (54)$$

<sup>7</sup> We could also study the Taylor model under the assumption that firms that set their price in the same period have initial conditions that involve heterogeneous capital. Under this assumption, there would be multiple prices chosen in the same period. However, as long as the size of the initial state was manageable, it would be feasible to analyze such a situation.



### Investment and Labor Demand

Here, for convenience, we collect the equations that were stated in Section 1 for the general model and the equations for optimal capital accumulation from the Calvo model. We express these equations in a form specific to the Taylor model. The technology is

$$y_{j,t} = k_{j,t}^\alpha (A_t h_{j,t})^{1-\alpha}. \quad (55)$$

Adjustment costs for the capital stock are

$$x_{j,t} = k_{j,t} G(k_{j+1,t+1}/k_{j,t}). \quad (56)$$

The first-order condition for next period's capital stock depends on the stage of the price cycle that a firm is in. To simplify notation, let " $j + i$ " denote  $(j + i) \bmod (J - 1)$  for  $j = 0, 1, \dots, J - 1$ . For example for  $j = J - 2$  and  $i = 3$ ,  $j + i = 2$ . The rewritten first-order condition (24) for next period's capital stock is then

$$G' \left( \frac{k_{j+1,t+1}}{k_{j,t}} \right) = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left\{ \frac{k_{j+2,t+2}}{k_{j+1,t+1}} G' \left( \frac{k_{j+2,t+2}}{k_{j+1,t+1}} \right) - G \left( \frac{k_{j+2,t+2}}{k_{j+1,t+1}} \right) + u_{j+1,t+1} \right\} \right]. \quad (57)$$

Real profits in period  $t$  for firm  $j$  are given by

$$d_{j,t} = p_{j,t} y_{j,t} - w_{j,t} h_{j,t} - x_{j,t}. \quad (58)$$

The marginal cost of production is

$$s_{j,t} = \frac{w_{j,t} h_{j,t}}{(1 - \alpha) y_{j,t}}. \quad (59)$$

## 4. RESULTS FOR THE TAYLOR MODEL

In this section we present results describing how the behavior of the Taylor model with firm-specific capital varies with the steady-state inflation rate around which it is linearized. We follow Sveen and Weinke's (2005) analysis of the Calvo model with firm-specific adjustment costs and zero steady-state inflation. First, we report on the range of parameters for the monetary policy rule and adjustment costs for which we can find unique RE equilibria. This range is sensitive to the steady-state inflation rate: higher inflation rates reduce the set of parameters for which there is a unique RE equilibrium. Next, we compare impulse response functions to a productivity shock for zero and moderate inflation. They differ, but not dramatically.

The model is parameterized as follows. We interpret a period as a quarter, and set the discount factor,  $\beta = 0.99$ ; the risk aversion parameter,  $\sigma = 2$ ; the inverse labor supply elasticity,  $\nu = 1$ ; the capital depreciation rate,  $\delta = 0.03$ ; and the capital income share,  $\alpha = 0.36$ . This is a standard parameterization.

We set the investment adjustment cost parameter,  $\epsilon_\psi = 3$ , as in Woodford (2005). Based on evidence from aggregate data, Eichenbaum and Fisher (2005) suggest that this value represents a lower bound for adjustment costs. Around a zero-inflation steady state, there is no need to specify the function  $G(\cdot)$  beyond the two parameters,  $\delta$  and  $\epsilon_\psi$ . Around steady states with nonzero inflation however, it is necessary to specify the entire function. We use

$$G(x) = \left( \delta - \frac{1}{1 + \epsilon_\psi} \right) + \frac{x^{1+\epsilon_\psi}}{1 + \epsilon_\psi}, \quad (60)$$

which satisfies the desired properties  $G(1) = \delta$ ,  $G'(1) = 1$  and  $G''(1) = \epsilon_\psi$ .

### Equilibrium Determinacy

A good monetary policy rule should imply a unique RE equilibrium. If the RE equilibrium is not unique, then at any point in time several different equilibrium time paths for current and future outcomes are possible. In other words, the equilibrium is indeterminate. In this situation the path that is expected to be chosen will occur, but many can be chosen. The choice of equilibrium path then may depend on random shocks that are not fundamental to the economy, that is, they do not constrain the set of resource-feasible allocations in the economy. In these “sunspot” equilibria self-fulfilling expectations that coordinate on the nonfundamental shocks introduce unnecessary fluctuations into the economy.<sup>8</sup> Since the representative agent is risk-averse, she will prefer a smooth consumption path relative to the same smooth consumption path with some added mean zero random fluctuations. This means that, in general, “sunspot” equilibria are sub-optimal, and a good monetary policy should not give rise to equilibrium indeterminacy.

Taylor (1993) proposed a monetary policy rule of the form  $f_\pi = 1.5$  and  $f_y = 0.125$  based on the outcomes of model simulations.<sup>9</sup> This policy rule reflects the Taylor principle that monetary policy should increase nominal interest rates more than one-for-one for any increase of inflation. In basic sticky-price models with reasonable specifications of price rigidity and without capital, this principle will, in general, imply a unique RE equilibrium. Sveen and Weinke (2005) evaluate the role of the policy parameter,  $f_\pi$ , and the degree of price stickiness,  $\phi$ , for the existence of unique RE equilibria in the Calvo model with firm-specific capital. They show that as the degree of price stickiness increases, the set of policy parameters for which there is local uniqueness becomes smaller. For the Taylor model we provide an analog to

<sup>8</sup> For a textbook treatment of sunspot equilibria, see, for example, Farmer (1993).

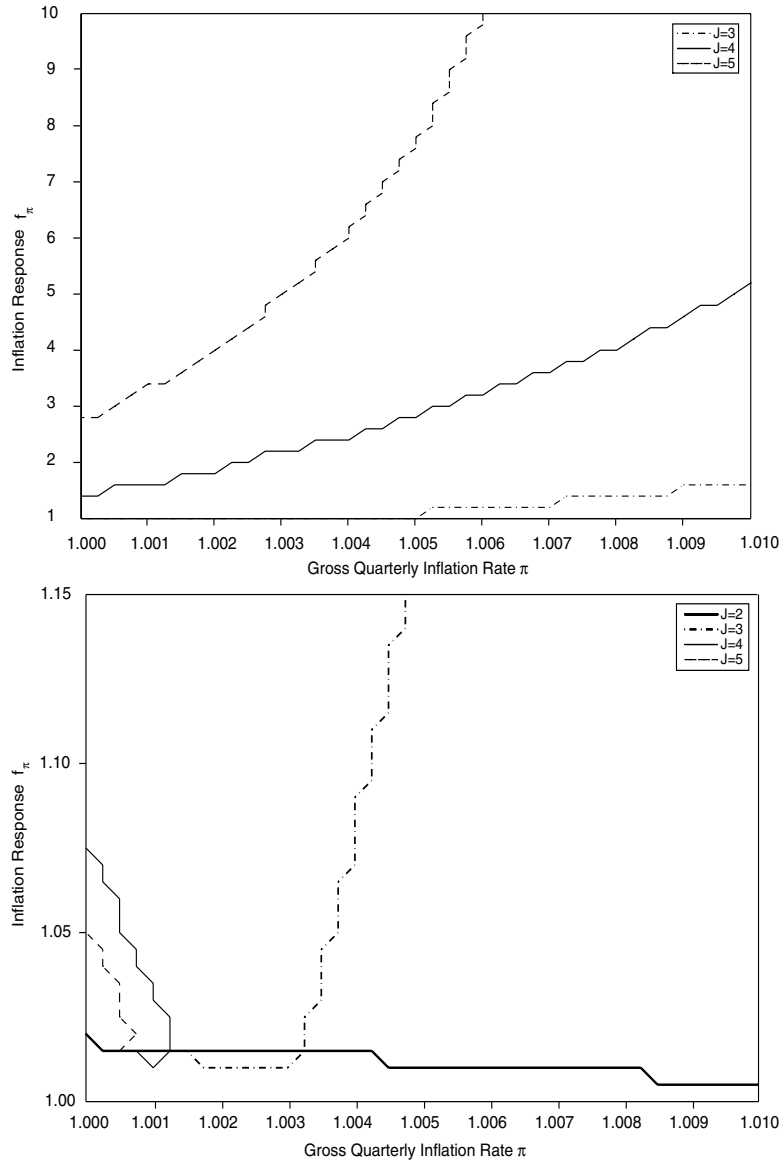
<sup>9</sup> Taylor (1993) writes the policy rule for annual data, thus his  $f_y = 0.5$  coefficient on output deviations translates to  $0.125 = 0.5/4$  in our quarterly model. Taylor’s proposed policy rule has also spawned an empirical literature that tries to estimate whether actual monetary policy conforms to some version of this policy rule, for example, Clarida, Galí, and Gertler (2000).

their results (price stickiness is now represented by  $J$ ). We also study the impact of the steady-state inflation rate,  $\pi$ , and investment adjustment costs,  $\epsilon_\psi$ , on equilibrium indeterminacy. We find that local uniqueness becomes less likely for higher inflation rates. Depending on the degree of price stickiness, high or low values of the adjustment cost parameter  $\epsilon_\psi$  can lead to indeterminacy.

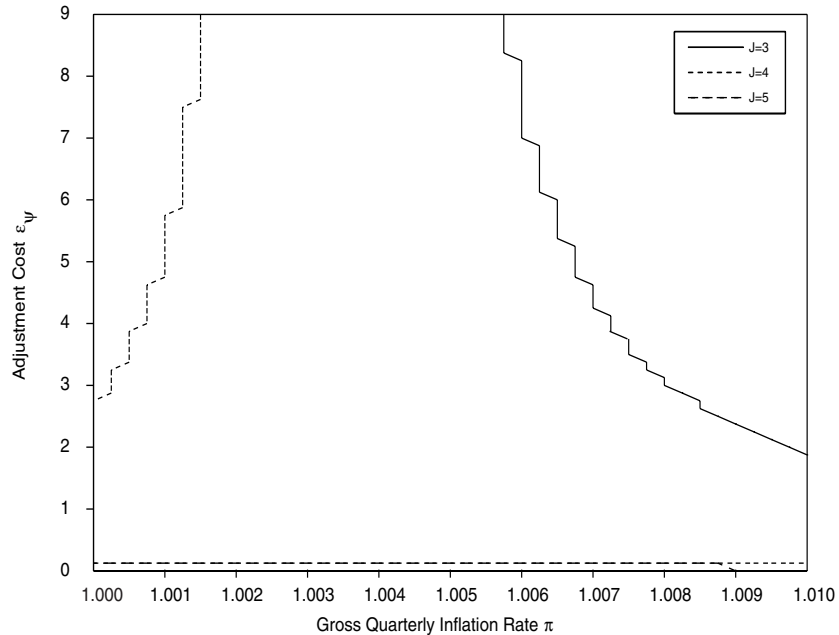
In Figure 1, we plot several graphs in  $(\pi, f_\pi)$ -space that represent the border between indeterminacy and uniqueness for a policy rule that does not respond to output,  $f_y = 0$ . We present this information in two panels because for very low values of  $f_\pi$ , it is not possible to convey the relevant information unless the  $f_\pi$ -axis scale is very fine. The inflation rate,  $\pi$ , is the rate of price change from one period to the next, and since a period represents a quarter, a gross inflation rate of 1.01 represents a 4 percent annual inflation rate. Each graph corresponds to a different value of  $J$ . In the top panel of Figure 1, which corresponds to relatively high values of  $f_\pi$ , the region of equilibrium indeterminacy (uniqueness) for an economy with price stickiness,  $J$ , is between the graph and the southeast (northwest) corner of the figure. There is no graph for  $J = 2$  in the top panel because uniqueness holds everywhere in the figure when  $J = 2$ . The bottom panel, corresponding to low values of  $f_\pi$ , is less straightforward: for  $J = 2$  there is indeterminacy below the graph; for  $J = 3, 4$  and  $5$  there is indeterminacy generally below and to the right of the graphs.

We find that for moderate steady-state inflation, if prices are fixed for more than two periods then policy needs to respond to inflation significantly more than one-to-one in order for the RE equilibrium to be unique. First, for all values of  $J$  and  $\pi$  that we consider, equilibrium is indeterminate if  $f_\pi$  is less than approximately 1.01 (the precise number varies with  $J$  and  $\pi$ ), as seen in the lower panel of Figure 1. In contrast, for the Calvo model with zero inflation, Sveen and Weinke (2005) find that there is a neighborhood of  $f_\pi = 1$  such that equilibrium is unique. Second, for fixed degrees of price stickiness,  $J > 2$ , the policy response  $f_\pi$  required to maintain a unique equilibrium can become quite large as we increase the steady-state inflation rate, as seen in the upper panel of Figure 1. This occurs even though the steady-state inflation rates that we consider are moderate, less than 4 percent per year. For example, if prices are fixed for three periods, around a zero-inflation steady state there is a unique equilibrium if  $f_\pi \gtrsim 1.02$ ; in contrast, around a 4 percent inflation steady state there is a unique equilibrium only if  $f_\pi \gtrsim 1.73$ . The sensitivity to steady-state inflation becomes more extreme for higher degrees of price stickiness. If prices are fixed for four periods, around a zero-inflation steady state there is a unique equilibrium if  $f_\pi \in \{(1.02, 1.074) \cup (1.47, \infty)\}$ ; in contrast, around a 4 percent inflation steady state there is a unique equilibrium only if  $f_\pi \gtrsim 5.29$ . Finally, for a given steady-state inflation rate, the region of

**Figure 1 The Monetary Policy Response to Inflation and Equilibrium Indeterminacy**



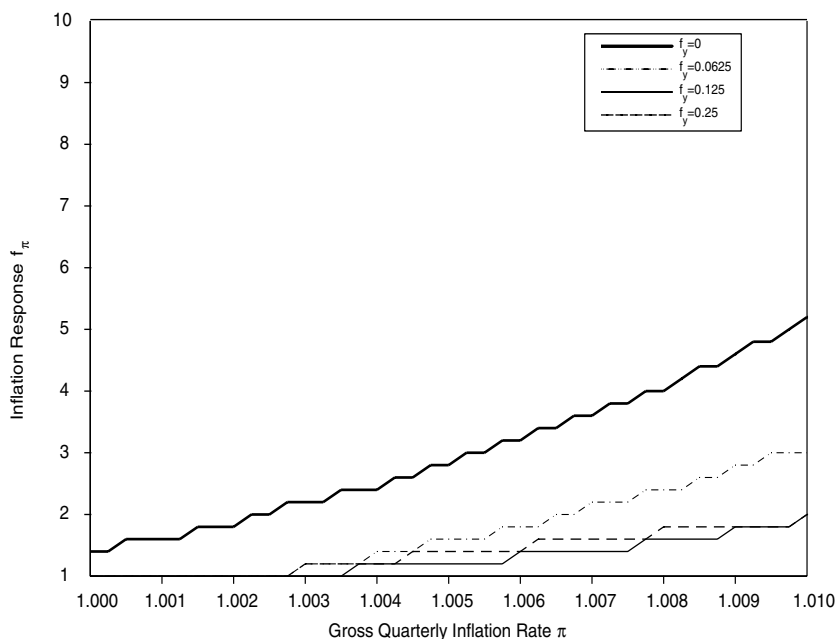
indeterminacy is increasing in the degree of price rigidity. This is consistent with Sveen and Weinke (2005, Figure 1).

**Figure 2 Investment Adjustment Costs and Equilibrium Indeterminacy**

For steady-state inflation rates that are even moderately high, the RE equilibrium tends to be indeterminate for a wide range of values of the adjustment cost parameter,  $\epsilon_\psi$ , but the precise relationship is sensitive to the degree of price stickiness. In Figure 2 we graph the borders between indeterminacy and uniqueness in  $(\pi, \epsilon_\psi)$ -space for different values of price stickiness  $J$  and a policy rule with  $f_\pi = 1.5$  and  $f_y = 0$ . For parameter combinations between a graph and the left (right) border of the figure, the RE equilibrium is locally unique (indeterminate) for  $J = 3$  and  $J = 4$  (there is also a region of uniqueness near  $\epsilon_\psi = 0$  for  $J = 4$ ). For  $J = 5$  there is indeterminacy (uniqueness) above (below) the graph. For  $J = 2$  there is uniqueness across the entire figure. For  $J = 3$  and  $J = 4$  the region of indeterminacy is increasing in the steady-state inflation rate. However, as the inflation rate increases, for  $J = 3$  indeterminacy first appears at high values of  $\epsilon_\psi$ , whereas for  $J = 4$  indeterminacy first appears at low values of  $\epsilon_\psi$ .

Sveen and Weinke (2005) argue that if a monetary policy rule responds not only to the inflation rate but also to output, then it is more likely that the RE equilibrium is unique. Indeed the Taylor rule (1993) specifies the coefficient on output as 0.125. In Figure 3 we graph the borders between indeterminacy and uniqueness in  $(\pi, f_\pi)$ -space for different values of the coefficient on output

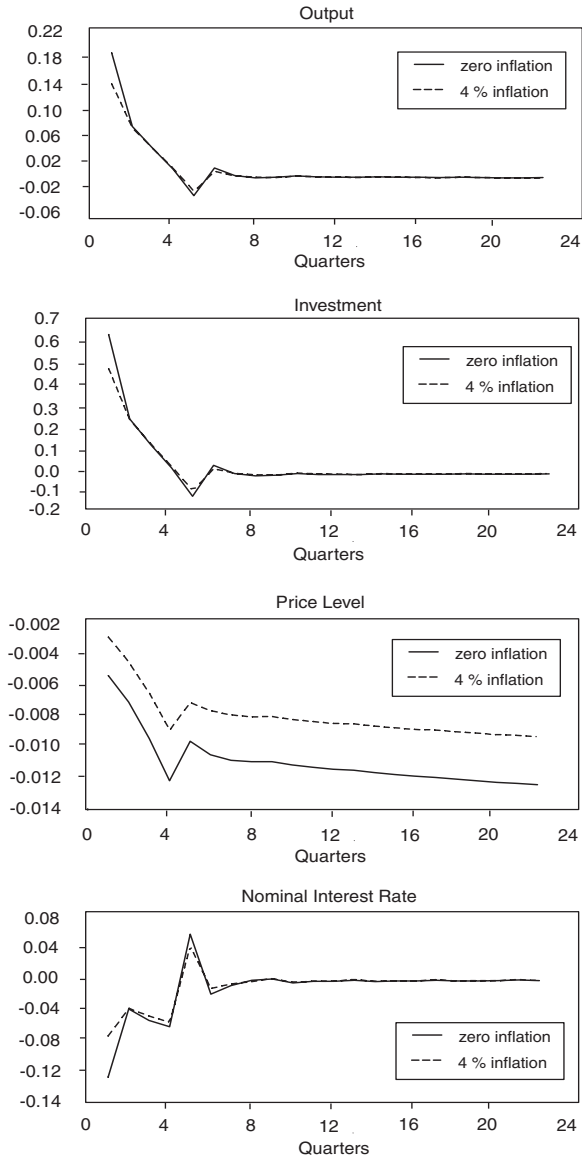
**Figure 3 The Monetary Response to Inflation and Output, and Equilibrium Indeterminacy**



in the policy rule  $f_y$  and fixed price stickiness  $J = 4$ . For parameter combinations between a graph and the left (right) border of the figure, the RE equilibrium is locally unique (indeterminate). Again, as the steady-state inflation rate increases, it becomes more likely that the RE equilibrium is not locally unique. For fixed steady-state inflation, the RE equilibrium is unique if the policy response to output is sufficiently large. This confirms the findings of Sveen and Weinke (2005). Note, however, that even for moderate steady-state inflation, it takes a large coefficient on output to generate determinacy in a rule that includes the standard Taylor coefficient,  $f_y = 0.125$ , on output. For example, for annual inflation of 4 percent (corresponding to  $\pi = 1.01$  in Figure 3), the coefficient on inflation needs to be greater than 2 in order to maintain a unique RE equilibrium. This is substantially more than the 1.5 value suggested by Taylor.

The overall message of these figures is that when the average inflation rate is even moderately high—say, above 3.5 percent annually—the coefficient on inflation must be large relative to conventional values such as Taylor’s 1.5 in order to generate a unique RE equilibrium.

**Figure 4 Impulse Response to Productivity Shock**



**Model Dynamics**

Figure 4 plots the response of several of the model’s aggregate variables to a white noise productivity shock. We set  $J = 4$  and  $f_{\pi} = 5.5$ . The solid lines correspond to a steady state of zero inflation, and the dashed lines correspond

to a steady state of 4 percent annual inflation. The responses to a productivity shock differ somewhat across very low and moderate inflation, but the differences are not dramatic, and they essentially disappear after the impact period. Given our findings about indeterminacy in Figures 1 and 2, it may seem surprising that the impulse responses do not differ more across steady-state inflation rates. There is, however, a good explanation for this. Unlike a crossing from uniqueness to nonexistence, a crossing from uniqueness to multiplicity need not be “foreshadowed” by large changes in the model’s dynamics. As we change a model’s parameters and uniqueness disappears, the solution we were tracking does not vanish—it is simply complemented with other solutions.

## 5. CONCLUSIONS

Svein and Weinke (2004) and Woodford (2005) have made important contributions in showing how one can linearly approximate the Calvo sticky-price model when capital is tied to the individual firm. Their work shows that capital adjustment costs at the firm level are complementary to price stickiness in generating a small coefficient on marginal cost in the New Keynesian Phillips curve. Around a steady state with nonzero inflation, it is not (yet) known how to approximate the Calvo model with firm-level investment; in such a steady state there would be heterogeneity in both prices and capital stocks. Much recent empirical work on the NK Phillips Curve has used data which is inconsistent with the zero-inflation approximation, so we would like to have some means of evaluating the generality of results from the zero-inflation case. In the Taylor sticky-price model it is straightforward to incorporate firm-specific capital even with nonzero steady-state inflation. Comparing zero- and moderate (4 percent) rates of steady-state inflation, one finds that *if there is a locally unique equilibrium*, quantitatively the model’s dynamics are not very sensitive to the rate of inflation. This is consistent with the work of Ascari (2004), who finds that the dynamics of the basic Taylor model (i.e., without firm-specific capital) are relatively insensitive to average inflation, in comparison to the Calvo model. However, we find that the range of parameter values for which the model has a locally unique equilibrium is extremely sensitive to even small changes in steady-state inflation—for example going from zero to 4 percent annual inflation causes a dramatic increase in the size of the parameter space for which there is local indeterminacy. The ability to deal with nonzero inflation in the Taylor model points toward the value of conducting empirical work on the New Keynesian Phillips curve in the Taylor model framework. See Guerrieri (forthcoming) for an important step in this direc-



tion.<sup>10</sup> However, the sensitivity of the local equilibrium uniqueness to the average inflation rate presents obstacles to further empirical progress.

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<sup>10</sup>Cogley and Sbordone (2005) is an important example of empirical work on the Phillips curve that allows for the possibility of nonzero steady-state inflation. They use a Calvo model with firm-specific capital but without firm-specific investment.

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## REFERENCES

- Ascari, Guido. 2004. "Staggered Prices and Trend Inflation: Some Nuisances." *Review of Economic Dynamics* 7: 642–667.
- Blanchard, Olivier J. and Nobuhiro Kiyotaki. 1987. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77 (4): 647–66.
- Calvo, Guillermo. 1983. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics* 12: 383–98.
- Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics* 115 (1): 147–80.
- Cogley, Timothy and Argia M. Sbordone. 2005. "A Search for a Structural Phillips Curve." Federal Reserve Bank of New York Staff Report 204.
- Coenen, Guenter and Andrew Levin. 2004 "Identifying the Influences of Real and Nominal Rigidities in Aggregate Price-Setting Behavior." ECB Working Paper 418.
- de Walque, Gregory, Frank Smets and Raf Wouters. 2004. "Price Setting in General Equilibrium: Alternative Specifications." Manuscript.
- Dixit, Avinash K. and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67 (3): 297–308.
- Eichenbaum, Martin and Jonas Fisher. 2004. "Evaluating the Calvo Model of Sticky Prices." NBER Working Paper 10617.
- Farmer, Roger E.A. 1993. *The Macroeconomics of Self-Fulfilling Expectations*. MIT Press: Cambridge MA.
- Galí, Jordi and Mark Gertler. 1999. "Inflation Dynamics: A structural econometric analysis." *Journal of Monetary Economics* 44 (October): 195–222.
- Green, Edward J. 2005. "A Review of Interest and Prices: Foundations of a Theory of Monetary Policy." *Journal of Economic Literature* 43 (March): 121–34.
- Guerrieri, Luca. Forthcoming. "The Inflation Persistence of Staggered Contracts." *Journal of Money Credit and Banking*.
- Sbordone, Argia M. 2002. "Prices and Unit Labor Costs: a new test of price stickiness." *Journal of Monetary Economics* 49 (March): 265–92.

- Svein, Tommy and Lutz Weinke. 2004. "Pitfalls in the Modelling of Forward-Looking Price Setting and Investment Decisions." Central Bank of Norway Working Paper 2004/1.
- \_\_\_\_\_. 2005. "New Perspectives on Capital, Sticky Prices, and the Taylor Principle." *Journal of Economic Theory* 123 (July): 21–39.
- Taylor, John B. 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88: 1–22.
- \_\_\_\_\_. 1993. "Discretion versus Policy Rules in Practice," Carnegie-Rochester Conference Series on Public Policy 39: 195–214.
- Wolman, Alexander L. Forthcoming. "The Frequency and Costs of Individual Price Adjustment." *Managerial and Decision Economics*.
- \_\_\_\_\_. 2004. "The Optimal Rate of Inflation with Trending Relative Prices." Federal Reserve Bank of Richmond manuscript.
- \_\_\_\_\_, and Elise A. Couper. 2003. "Potential Consequences of Linear Approximation in Economics." Federal Reserve Bank of Richmond *Economic Quarterly* 89 (1): 51–68.
- Woodford, Michael. 2003. *Interest and Prices*. Princeton University Press. Princeton, NJ.
- \_\_\_\_\_. 2005. "Firm-Specific Capital and the New-Keynesian Phillips Curve." *International Journal of Central Banking*. Forthcoming.