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# External vs. Internal Learning-by-Doing in an R&D Based Growth Model*

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**Abstract**

In an economy where growth is determined by the interaction of R&D and learning-by-doing (LBD), changes of factors that stimulate either one of these activities affect growth differently than in an economy where growth is determined by either R&D or LBD alone. In particular, when firms anticipate that R&D for new types of goods threatens the future efficiency gains which they derive from LBD, a more efficient learning process or a larger workforce might reduce rather than increase the growth rate on a balanced growth path.

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## 1. Introduction

Learning-by-doing, the process by which current and past production experience permanently increases productivity, has been portrayed as a prominent engine of growth in the literature.[1] This paper questions some important conclusions implied by typical formulations of learning-by-doing (LBD) in that literature, and shows that these implications are reversed under the formulation proposed here. In existing models LBD has growth effects through externalities which increase without bounds the effective productivity of factors of production or the range of producable goods. Moreover, because these effects appear as pure externalities, producers in these models do not take into account the impact of their own current production levels on future LBD-induced benefits. Such models imply that the steady state growth rate increases with the level of resources in the economy, ("scale effects"), and with the speed of learning.

While there is substantial evidence that LBD increases productivity, that same evidence shows that its potential for enhancing growth is of a completely different nature than that assumed in the literature. First, measured productivity improvements due to LBD for any particular good are bounded. Second, these productivity improvements are mostly limited to the firm producing the product and are internalized by the firm. Third, there is no strong evidence that production experience with existing products is directly transferable to new types of products. Accordingly, the impact of LBD on new products cannot be simply described by way of reduced unit production cost. We show that in a growth model, which incorporates a richer version of LBD based on these observed features, the standard results are overturned. In particular, for an empirically plausible parameterization of the model

[1] See, for instance, Romer (1986), Stokey (1988), Lucas (1993), and Young (1993).

the growth rate declines when productivity is more sensitive to experience, and when the resource base of the economy increases.

The benefits associated with LBD can be classified into two broad categories. Although distinct, both depend on cumulative experience with existing goods and production practices. First, through repetitive and routine production experience, LBD reduces production costs and generates minor product improvements. These efficiency gains are bounded and are relevant for the production of a new good in a particular production site. Such gains are well documented in the management literature on LBD.[2] Because this mechanism implies only bounded productivity improvements it cannot generate sustained growth. A second benefit attributed to LBD, which is capable of sustaining growth, is associated with the research for and the development of new products and production technologies. As bottle necks in current production practices are identified, the ways in which customers use the product become known, and the product's main limitations and deficiencies are revealed, innovators are better able to come up with new products and better production technologies. However, since re-engineering, labor mobility, and public discussions of customers' needs are permanent features of market economies, the knowhow which turns up to be useful for developing new products can hardly be appropriated by current producers.[3]

The fact that potentially proprietary efficiency gains from increased production levels are inseparably bundled with the creation of R&D opportunities which cannot be appropriated creates a complicated interaction between LBD, R&D, and production decisions. We study

[2] See, for instance, Argeote, Backman, Epple (1990), Bahk and Gort (1993), and references therein.

[3] Note that this externality is very different from the usual assumption that LBD directly expands the range of products which can be produced. In particular the second LBD benefit described implies that the potential for successful R&D increases as more experience cumulates with existing goods, but not that the cumulated experience directly determines the rate of new product introduction.

this problem in a modified version of the quality ladder growth model of Grossman and Helpman (1991) and Aghion and Howitt (1992). The modification takes the form of adding an experience measure of quality to each product, which is tied to its production volume. Specifically, there are two levels of effective quality associated with each product. New goods start with the low-experience quality, and move up to the higher level of quality only after enough experience has been accumulated with them. The high-experience status of a good is more profitable to its producer due to greater consumer demand or reduced unit cost. R&D for new products can only be performed in those industries whose products have already attained the high-experience status. This description captures the two distinct types of LBD benefits, while maintaining their inseparable bundling.

The R&D-enabling benefit of a high experience status attained by any product is modeled as an externality which cannot be appropriated by the current producer of that good. We consider three alternative assumptions concerning the externalities associated with efficiency gains generated by LBD. Specifically, the "learning rate", the rate at which existing products attain the high-experience status, is described as: (i) completely exogenous, (ii) depending on aggregate production level, (a production externality), (iii) an internalized production benefit, the rate of which depends only the production level of that particular good. Profit maximizing production and R&D decisions jointly determine the distribution of experience across goods in the economy and the extent of R&D activities for developing new goods. Growth, arising from the introduction of new types of goods, is affected by both.

The key role of the endogenous distribution of experience across goods in the economy in determining the growth rate, and the considerations that enter producers' decision about the pace of their own experience accumulation, set us apart from most of the literature on growth through R&D for new products. These models assume that firms do not internalize the effects of LBD. That is, producers in those models fail to recognize that higher production volume today implies lower production costs tomorrow, and also a greater likelihood of a new product being introduced that might preclude them from enjoying those efficiency gains.

In the classic formulation of these externalities, general labor productivity increases over time at a rate which is proportional to aggregate output. Most models that explicitly consider the distinction between existing goods and goods to be developed also maintain the strict externality associated with LBD. For instance, Stokey (1988) does not make a distinction between the development of new goods and the improvement of existing goods. Rather, she assumes that higher level of output today reduces the unit cost of both existing goods as well as goods to be developed, and that this benefit is external to all firms. In Young (1993), experience with existing goods reduces their unit cost, and that cost reduction is external the firms. The potential impact of experience on R&D for new products is assumed away altogether. Parente (1994) describes adoption decisions of new technologies by firms, but the rate at which they gain experience and become more efficient in using those new technologies is independent of their production volume. Lucas (1993) recognizes that LBD reduces both the unit cost of existing goods and goods in development, but does not incorporate these effects in firms' production decision. Consequently, these models only partially capture the forces at play when the learning technology improves, R&D costs decline, or the labor force size increases.

It turns out that in our setup, the magnitudes or even the sign of the growth effects of such changes in the environment are very sensitive to the benefits and the speed of acquiring the high-experience status. The vast empirical literature on LBD is almost exclusively confined to the efficiency benefits that accrue from repetitive production of the same good, using the same method, in the same facility. Very few works estimate the extent to which LBD benefits spill over across production facilities and successive generations of products.[4] When we parametrize our model in light of the evidence in the latter and let producers internalize the efficiency benefits of LBD, the increased risk of loosing future benefits from their current LBD activities tends to dominate other growth enhancing influences of the exogenous changes considered.

In Section 2 we compile some of the empirical evidence on the characteristics of LBD, in terms of types of benefits, their magnitude, and the extent to which they are external. In section 3, we present our modified version of an R&D based growth model with endogenous learning. In section 4 we show that while the usual growth implications of increased work force, lower R&D costs, or faster learning rate obtain when learning is exogenous, (i.e., independent of output volume), it is no longer possible to determine analytically the impact of those same changes with endogenous learning. Because the analysis with endogenous learning is so complicated, we turn in section 5 to a numerical computation of the equilibrium in a version of the model parametrized in light of the evidence compiled in section 2, which fits the characteristics of experience spillovers across different products and production facilities. The numerical solution shows that producers' responses to improved learning technology, reduced R&D costs, and larger work force size result in a lower steady state growth rate.

[4] Irwin and Klenow (1994), Bahk and Gort (1993), Argote, Backman and Epple (1990).

## 2. Characteristics of Learning-by-Doing

LBD refers mostly to the process by which unit production costs are reduced as experience is accumulated. This reduction in production costs is present even after controlling for changes in capital investment and other purchased inputs. Thus, in its original meaning, LBD captures the enhanced efficiency attributed to increased familiarity with the routine activities performed by the firm. The significance and magnitude of LBD effects in the context of a specific production process has been noted in the engineering literature for many decades: one of the earliest references is Wright (1936) on the labor costs of airplane frames. Recently there is also a growing recognition that a distinction ought to be made between LBD processes associated with a specific product, and those associated with the plant or the organization. For example, Bahk and Gort (1993) decompose learning into three types of LBD: labor learning which captures the increase in manual skills obtainable through repetitive tasks; capital learning which refers to engineering information accumulated over time in the use of particular capital equipment; and organizational learning, the most elusive concept of the three, referring to managerial, marketing, and general types of knowledge that accumulate in an organization over time. Most of the empirical work on LBD has focused on the first kind, where it is measured as reduction in average unit production cost as a function of cumulative production volume.[5]

The model we develop in this paper is too stylized to allow us a distinction between all these different types of learning. Yet all three are important to the process of introducing new goods and technologies, and all three create possible external and spillover influences.

[5]Some attempts to measure cumulative experience by cumulative past investment, or simply elapsed time, have also been considered, (Sheshinski 1967).

From the point of view of our theory three characteristics of LBD are important: (i) the learning rate, measuring the elasticity of unit cost, (equivalent in our model to product quality), with respect to cumulative output; (ii) the learning scope, measuring the overall efficiency gain to be attainable through LBD, and (iii) the extent to which externalities are present in LBD.

A stylized fact of the empirical work on LBD is that the learning rate is about twenty percent, that is a one percent increase in cumulative output reduces cost by about 0.2 percent. Learning elasticities of this magnitude on specific products have been found in early empirical work by Searle (1945), Rapping (1965), Hirsch (1952), and Cole (1958). Recently, Irwin and Klenow (1994) in their study of the production of DRAM chips in the semi-conductor industry have found a similar learning rate. So pervasive is this phenomenon, that it is referred to as the "learning curve", or even the "80% curve", reflecting the empirical regularity in manufacturing industries where the unit costs of the 2n$^{th}$ unit are 80% of those of the n$^{th}$, Alchian (1959). However the 80% learning curve is a stylized fact, and as such there are other studies indicating some variation in observed learning elasticities across various industries, or even within the same industry. Most of this empirical work is concerned with cost reductions for a new product within an existing production unit, for example Alchian (1963), Baloff (1966), Billon (1966). Learning elasticities of 50% or more are found by Argote, Backman and Epple (1990) in a study of world war II shipyards. At the other extreme Bahk and Gort's (1993) study of new manufacturing plants, (i.e. new production units), indicates considerably lower learning rates.

While learning curves imply continuous cost reductions as the cumulative production volume increases, there is ample evidence that these learning effects are bounded, or that learning eventually ceases, (Asher (1956), Conway and Schultz (1959), Baloff (1966, 1971), Hall and Howell (1985)). We refer to the maximum potential quality improvement attainable through LBD as the scope of learning. Empirical work on cost reductions for new products within existing production units finds that the scope for learning is large. For example, Searle (1945) documents a 50% reduction in man-hours input for the Liberty ships. However, the possibilities for cost reductions through LBD within new production units appear to be much smaller. For a panel of new manufacturing plants Bahk and Gort (1993) find that new plant start with around 80% of their full potential productivity.

Most of the aforementioned learning rates have been estimated relative to a production unit's own cumulative production experience with the same specific product or process. This type of learning, which can be thought of as *internal* learning, stands in sharp contrast to the type of *learning externality* exploited by Stokey (1988) and Young (1993), where economy-wide or industry *aggregate* cumulative experience determines the productivity of an individual producer at a point in time. In contrast with the persuasive empirical evidence on the presence and magnitude of internal learning, there is limited evidence for learning externalities of the variety considered by Stokey (1988) and Young (1993), (Argote, Backman and Epple (1990), Bahk and Gort (1993)).[6] We do not dispute that there is an important learning externality associated with LBD in the sense that new products are not introduced before some experience with existing products has accumulated, (in fact we incorporate such a feature in the model developed below). We claim, however, that the parameters governing this learning process may be quite different from those characterizing the enhanced efficiency

[6] Irwin and Klenow (1994) find some evidence for external effects but internal LBD appears to be quantitatively much more important.

learning which has been studied thus far.

The general characteristics of LBD that emerge from the above survey are: (i) learning has a significant effect on efficiency; (ii) learning increases as a function of production volume; (iii) the scope of learning is bounded; (iv) there is an important component to learning which is firm-specific, (v) the experience effect on the development of new goods is more modest than its impact on efficiency.

## 3. A Simple Model of Learning-by-Doing and R&D

Growth may originate through the development of new products or the improvement of existing products. We identify the first source of growth with R&D based growth, and the second source with LBD. Although the growth effects of LBD may be bounded, LBD can affect growth through its interaction with R&D. We model the interaction of LBD with purposeful R&D in the "quality ladders" structure proposed by Grossman and Helpman (1991). Consequently, we follow their presentation of the basic framework, appropriately modified, in order to introduce our notation. The environment is one with an infinite horizon and with a continuum of goods at each point in time. We will limit our analysis to the study of balanced growth paths where all goods are treated symmetrically. Accordingly, we will drop the time and goods' index whenever confusion does not arise. We will first describe the household's dynamic optimization problem, then the production structure of the economy, and conclude with a definition of the balanced growth path.

### 3.1. The Household

There is a representative agent with preferences over the continuum of goods indexed by $\omega \in [0,1]$

$$\int_0^\infty e^{-\delta t} \int_0^1 \log[x(\omega,t)q(\omega,t)]d\omega\ dt, \tag{1}$$

where $x(\omega,t)$ and $q(\omega,t)$ are, respectively the quantity and quality of good $\omega$ consumed at time $t$, and $\delta > 0$ is the rate of time preference. The household's optimization problem can be separated in two stages. First, the household maximizes flow utility subject to the flow budget constraint

$$\max_x \int_0^1 \log[x(\omega,t)q(\omega,t)]d\omega \quad \text{s.t.} \quad \int_0^1 x(\omega,t)p(\omega,t)d\omega = E(t), \tag{2}$$

where $p(\omega,t)$ is the price of a unit of good $\omega$ at time $t$, and $E(t)$ is total expenditures on goods at time $t$. This implies a unit elastic demand function for each good

$$x(\omega,t) = E(t)/p(\omega,t). \tag{3}$$

Secondly, the household chooses an expenditure path which maximizes lifetime utility (1) subject to the life-time budget constraint

$$\int_0^\infty e^{-R(t)}E(t)dt \leq A(0), \tag{4}$$

where $A(0)$ is the agent's initial wealth, and $R(t)$ is the cumulative interest rate up to time $t$. An optimal expenditure path satisfies the Euler equation

$$\dot{E}(t)/E(t) = r(t) - \delta \tag{5}$$

and $r(t) = \dot{R}(t)$. On a balanced growth path the interest rate is constant $r(t) = \delta$, so that $E(t) = E$ for all $t \geq 0$.

### 3.2. Production, Learning-by-Doing, and R&D

The quality of a good improves over time for two reasons. First, goods of new basic qualities are introduced at each point in time via an R&D process, as in Grossman and Helpman (1991). Second, once a producer introduces a new basic quality, that quality improves as the producer becomes more experienced in the production of the good. We assume that this improvement through learning is bounded, and that the basic quality of a good cannot be upgraded before its producer has acquired some experience. We now describe the mechanism by which LBD proceeds, how experience affects production, and how it interacts with R&D.

Production of goods is constant returns to scale with labor as the only input. In particular, $x$ units of labor produce $x$ units of any good, of any variety, regardless of experience. Superior quality, embedded in higher generation of basic quality or more experience, is reflected in *effective* quality rather than direct production costs.[7]

Total or effective quality of any good is given by $q(\omega, t) = e(\omega, t) \cdot \lambda^{m(\omega,t)}$, where the

[7] Although the empirical literature emphasizes reduced unit cost as the main benefit of LBD, we describe that benefit as an improved quality in order to maintain the similarity with the quality ladder model of Helpman and Grossman (1991). These two descriptions are completely equivalent in our model, (see section 3.4).

integer $m(\omega, t)$ is the *basic quality* of the good, and $e(\omega, t)$ is the *experience* level with that particular good. New basic qualities are introduced successively, and each new basic quality is $\lambda$ times as good as the previous basic quality, $\lambda > 1$. To capture the dynamics of experience accumulation it suffices to assume that experience can take on two values, $\{e_1, e_2\}$, where $e_1 = \epsilon \in (0, 1)$, and $e_2 = 1$. The process by which the experience factor increases from $e_1$ to $e_2$ is *learning*. Learning is *by doing* in the sense that only firms which actually produce and sell a product can benefit from experience enhancement. Each producer introducing a new good begins with the low experience level, $e_1$, and $1 - \epsilon$ denotes the *scope* of learning. "Experience enhancement" from $e_1$ to $e_2$ follows a Poisson process with a *learning rate* $\alpha \geq 0$ per unit of time. We consider three versions of the learning process: a firm's learning rate may be (1) exogenous, (2) endogenous, but independent of the firm's production, and (3) endogenous and dependent on the firm's production.

For the first case, a firm learns only when it produces, but the rate at which it learns is constant. Although this description is inconsistent with the evidence surveyed in section 2, indicating that the rate of learning depends on the volume of production, this case allows us to highlight the implications of endogenous learning. In the second case considered, we assume that the learning rate increases with the total production volume by all inexperienced firms $X_1$, $\alpha = a(X_1)$. This learning externality reflects the idea that inexperienced producers work on similar problems, and solutions to these problems are partially transferable, see for example Young (1993).[8] Finally, for the third case we assume that the learning rate of a firm

[8] Alternatively we could have assumed that inexperienced firms learn from experienced firms and the learning rate depends positively on the total production volume of all experienced firms. Such a modification could be based on a tendency to "learn from those who do it right". This alternative approach yields essentially the same results for our analysis of the calibrated steady state in Section 5. Neither formulation affects the definition and characterization of a steady state.

depends on its own production volume, $\alpha = a(x_1)$, and the learning rate is an increasing and concave function of production $x_1$. This last case allows a firm to increase current production in order to enhance learning, thus trading off current for future profits.

Improved basic quality comes as a result of deliberate and costly R&D. The R&D technology is stochastic and displays constant returns to scale, and entry to the R&D sector is free. If the current producer in an industry is experienced, (has already achieved $e_2$), a flow R&D investment of $\rho > 0$ units of labor generates a *unitary* arrival rate of the next basic quality. Thus, $\theta\rho$ units of labor are needed to produce an arrival rate $\theta$ per time unit of a one-step improvement of the basic quality of a good. The current producer enjoys no inherent advantage over potential innovators in this R&D activity. Our assumption that a basic quality improvement can only be achieved in industries that have already achieved the high experience level reflects the spillover from production experience with existing goods to developing new ones.[9]

### 3.3. The Balanced Growth Path

The description of the balanced growth path is analogous to Grossman and Helpman (1991) with a minor modification for the third learning specification when the effect of production on learning is internalized. In all cases considered only the highest basic quality of each variety is actually produced and sold in equilibrium, since, with Bertrand price competition, the producer with the highest basic quality can capture the entire market by under-pricing

[9] A less extreme assumption is that the same R&D investment is more likely to succeed in industries that have already attained high-experience with the previous basic quality than it is in industries that have yet to reach that state. This, however would considerably complicate the analysis without affecting the nature of the results.

all other producers of the same good.

Let $m$ be the highest available basic quality of a particular good, and let $e_i$ be the experience level associated with the basic quality level. We refer to the producer of the highest basic quality of any good as the leader or incumbent. At any instant and for any good, the leader's experience can be either $e_1$ if she is inexperienced, or $e_2$ if she has already benefited from experience enhancement. The producer of the previous basic quality, $m-1$, is by assumption experienced, and is referred to as the *follower*. A leader with experience $e_i$ will choose a price $p_i$ such that producers of lower basic qualities are excluded. We obtain this price by considering the price per effective quality. Specifically, since the follower always has the high experience, $(e_2 = 1)$, and her marginal production cost is unity, a leader will choose a price which satisfies the constraint

$$\frac{p_i}{e_i \lambda^m} \leq \frac{1}{\lambda^{m-1}}, \text{ or } \quad p_i \leq \lambda e_i, \; i = 1, 2, \tag{6}$$

and the limit price $\lambda e_i$ does not depend on the basic quality $m$.[10] The flow profit for an incumbent with experience $e_i$ is:

$$\pi_i = x_i(p_i - 1) = E - x_i, \tag{7}$$

where $x_i = E/p_i$ is the quantity demanded of the good.

All incumbents enjoy continuously their flow profits, while inexperienced ones also stand the chance of experience enhancement, whereas experienced ones face the danger of being

[10] If the follower could be of either high- or low-experience, the leader's price would depend on both experience levels.

displaced.[11] Let $V_i$ be the capital value of an incumbent with experience $e_i$. The Bellman equations which define the capital values of incumbents on a balanced growth path are given by:

$$\begin{aligned} rV_1 &= \max_{x_1}\{E - x_1 + \alpha(V_2 - V_1)\} \\ &\quad \text{s.t. } x_1 \geq E/\lambda\epsilon \text{ and } \alpha = \begin{cases} \text{constant} & \text{if LBD is exogenous} \\ a(X_1) & \text{if LBD is external} \\ a(x_1) & \text{if LBD is internal} \end{cases} \\ rV_2 &= \max_{x_2}\{E - x_2 - \theta V_2\} \\ &\quad \text{s.t. } x_2 \geq E/\lambda \end{aligned} \tag{8}$$

where $X_i$ is the total quantity produced of all goods with experience level $e_i$. Note that since demand is unit-elastic profit is declining with higher production. When the learning rate $\alpha$ is independent of his own production an incumbent will therefore set the price equal to the limit price; this applies to the exogenous, and external LBD cases above.

Successful R&D generates temporary monopoly profits, and R&D will take place as long as the cost of R&D is equal to the expected capital value of the monopoly profits. The gain from devoting $\theta\rho$ units of labor to R&D is the capital value $V_1$ and it follows a Poisson process with arrival rate $\theta$. The expected gain is $\theta V_1$, and free entry into R&D implies

$$V_1 \leq \rho, \tag{9}$$

[11] Incumbents are not engaged in R&D because they do not have a particular advantage in R&D and they benefit less from an additional quality improvement. We assume free entry into R&D attempts to discover the next basic quality. To see that this implies no R&D by the current incumbent, note that with many potential enterants the probability of the next basic quality being found by one of the attempting enterants is unaffected by the incumbent's own R&D efforts. Moreover, free entry and constant returns to scale in R&D imply that potential entrants are at best breaking even on their R&D expenditures. Finally, since the incumbent gains less from discovering the next basic quality than a potential enterant does, we conclude that incumbents do not invest in R&D.

with equality if R&D is done.

An important part of the equilibrium involves how many leaders are (in)experienced. Let $\mu_i$ be the measure of industries with an $e_i$-incumbent, $\mu_1 + \mu_2 = 1$. This measure changes because of LBD, and because of R&D activities. The measure of inexperienced incumbents declines because some incumbents become experienced, and it increases because some experienced incumbents are replaced by inexperienced entrants. The rate of change of the measure of inexperienced incumbents is then:

$$\dot{\mu}_1 = -\alpha\mu_1 + \theta\mu_2. \tag{10}$$

On a balanced growth path $\dot{\mu}_1 = 0$, so that $\mu_2 = \alpha/(\theta + \alpha)$.

To close the model, we need to clear the labor market. This requires that aggregate labor demand equals the exogenous labor supply. Labor is used for R&D, and for production. Production in industries in which the incumbent has experience $e_i$ is $X_i = \mu_i x_i$, so that aggregate production demand for labor is $\sum_i \mu_i x_i$. R&D demand for labor is $\mu_2\rho\theta$ since R&D is done only against high experience incumbents. With $L$ denoting the exogenously fixed labor supply, we have

$$\mu_1 x_1 + \mu_2 (x_2 + \rho\theta) = L. \tag{11}$$

We are now in a position to define a balanced growth path for our economy.

**Definition 1.** *A balanced growth path is a set of endogenous variables,* $\{E, \theta, \mu_i, x_i, \pi_i, V_i\}$, *satisfying equations (3), (6), (7), (8), (9), (11), and (10) with* $\dot{\mu}_1 = 0$.

Growth in the utility index $u$ can only stem from basic quality improvements, since labor supply and labor productivity are constant. The contribution of good $\omega$ to the utility index is

$$\log[x(\omega,t)q(\omega,t)] = \log[E(t)] + m(\omega,t)\log(\lambda) \tag{12}$$

where we have used the definition of the limit price (6) and the demand function, (3). The utility flow index evolves according to

$$\log u(t) = \log(\lambda)\int_0^1 m(\omega,t)d\omega + \log\left[E(t)\right]. \tag{13}$$

On a balanced growth path $E(t) = E$. Thus growth in the utility index stems solely from *basic quality upgrading*, the first term in (13). Since the evolution of basic quality upgrading for each good $\omega$ is an independent stochastic process, the average quality index of basic quality across all goods at time $t$ is the cumulative number of basic quality increments in each good during the time interval $[0,t]$, $m(\omega,t)$, averaged across all goods. Assuming a law of large numbers applies, the cumulative number of basic quality upgrades in each good is given by the length of the period times the expected number of basic quality discoveries per unit of time. Consequently,

$$\int_0^1 m(\omega,t)d\omega = t\cdot(\theta\mu_2) \tag{14}$$

It follows that the growth rate of $u(t)$ on a balanced growth path is

$$g = \theta\mu_2\cdot\log(\lambda) \tag{15}$$

Note that bounded learning has no direct effect on the growth rate, although it does affect it through its impact on the endogenous R&D rate $\theta$, and the measure of experienced firms, $\mu_2$.

### 3.4. Cost Reduction vs. Experience Enhancement

We have described a simple model of LBD where more experience means that a firm's product is of higher quality. In our discussion of empirical evidence on LBD, more experience means that a firm produces the same product at a lower cost. In our framework these two formulations are almost equivalent. Assume that the effective quality of a product is always equal to the basic quality of the product, but that the production costs of producers differ. In particular assume that a producer is either inexperienced with labor input requirement $\gamma_1$ or experienced with input requirement $\gamma_2$, $\gamma_1 > \gamma_2$. Normalize $\gamma_2 = 1$ and $\gamma_1 = 1/\epsilon > 1$. The analysis now proceeds as before, and the only difference between the two formulations is that now all producers set the same price and sell the same quantity, independent of their experience.

## 4. Balanced Growth With R&D and Learning-by-Doing

From the analysis in the previous section, the balanced growth rate in our model is given by:

$$g = \theta\mu_2 \log(\lambda) = \frac{\theta\alpha}{\alpha + \theta} \log(\lambda)$$

With exogenous learning, ($\alpha$ being a fixed parameter), the growth rate increases with R&D intensity $\theta$. Moreover, because R&D can improve basic quality only of goods that have already attained high experience, the growth rate increases less than proportionately with the increase in $\theta$, so that $0 < \frac{\partial g}{\partial \theta}\frac{\theta}{g} < 1$.[12] Likewise, holding R&D intensity fixed, similar growth implications are generated by changes in the learning rate. However, these monotone relationships become more involved when both sources for quality improvement are jointly determined in equilibrium. In particular, if equilibrium learning through experience accumulation declines with factors that stimulate R&D then the impact of such factors on growth depends on relative magnitudes of opposing forces.

### 4.1. Exogenous Learning-by-Doing

Following Grossman and Helpman (1991) we proceed to solve the equilibrium in the model by obtaining two relationships in terms of the level of real expenditures $E$ and the R&D rate $\theta$. The first relation $(FE)$ is based on the free entry condition (9), and the second relation $(LL)$ is based on the labor market clearing condition (11). We focus on solutions to these two equations, $(E^*, \theta^*)$, given the exogenous learning rate $\alpha$, the scope of learning $\epsilon$, and the resource base, $L$.

In Figure 1 we graph the free entry condition and the labor market clearing condition.

$$\begin{aligned} E &\leq \rho(\delta+\alpha) \Big/ \left[\left(1-\tfrac{1}{\lambda\epsilon}\right)+\tfrac{\alpha}{\delta+\theta}\left(1-\tfrac{1}{\lambda}\right)\right] \quad \text{and } '=' \text{ if } \theta>0 \qquad (FE) \\ E &= \lambda\left[\tfrac{\alpha+\theta}{\alpha+\theta/\epsilon}L-\tfrac{\alpha\theta}{\alpha+\theta/\epsilon}\rho\right] \qquad (LL) \end{aligned}$$

The $(LL)$ curve is downward sloping: as the research intensity $\theta$ increases, total demand for labor increases, requiring a reduction in the level of real expenditures $E$ to bring the labor demand back to the given supply. The demand for labor increases for two reasons. First,

[12] In the model absent learning-by-doing, R&D applies to all goods in the economy and one gets the proportionality of the growth rate and R&D intensity, $\frac{\partial g}{\partial \theta}\frac{\theta}{g} = 1$, as in Grossman and Helpman (1991).

manufacturing labor demand increases because the share of inexperienced firms increases, and these firms produce more due to limit pricing, (lower quality implies lower price and larger quantity supplied). Second, although the share of experienced firms declines, the increase in research intensity is sufficient to raise total R&D demand for labor. The $(FE)$ curve is upward sloping: as research intensity increases, the capital value of being an incumbent declines, so that the level of real expenditures has to increase to allow $V_1$ to satisfy the free entry condition. Note that in the limit when learning is instantaneous, $(\alpha = \infty)$, the labor market clearing and the free entry condition coincide with Grossman and Helpman (1991) equilibrium conditions. It is immediate that slower exogenous learning, (lower $\alpha$), makes it more likely that the equilibrium does not involve positive growth.[13]

FIGURE 1 HERE

In the simple model with exogenous LBD the interaction between learning and R&D is limited: we can only study how learning affects R&D, but not vice versa. An analysis of the balanced growth path, however, reveals that even in such a simple environment the effects of learning on R&D can be ambiguous. For example, even though one would expect slower learning to reduce incentives to do R&D, (since experience enhancement following a success in R&D is further postponed into the future), total R&D efforts in the economy may actually increase with slower learning.

To see that, consider the effect of an increase in the learning rate on the demand for labor and the incentives to do R&D. First, a faster learning rate increases the share of

[13]Consider the extreme case where $\alpha = 0$, so that firms never learn, and both the $FE$ and the $LL$ curves become horizontal in this case. The share of experienced firms converges to zero, and R&D activities aimed only at experienced firms cease altogether.

experienced incumbents. Consequently, manufacturing demand for labor declines, but R&D demand for labor increases. If the research intensity is sufficiently high, then the second effect will dominate and the demand for labor increases. One can show that in Figure 1, an increase in the learning rate will turn the $(LL)$ curve clockwise around the point $(\theta = \theta_{LL}, E = \lambda \epsilon L)$, where $\theta_{LL} = (1 - \epsilon) L/\rho$. Second, a faster learning rate makes the profit flow less important relative to the capital gain for an inexperienced incumbent. Although profits for an inexperienced incumbent are lower than for an experienced incumbent, inexperienced incumbents are not subject to R&D competition. Thus for sufficiently high research intensity, the capital value of an inexperienced incumbent may actually fall following an increase in the learning rate. In terms of Figure 1, one can show that an increase in the learning rate will turn the $(FE)$ curve counter-clock wise around the point $(\theta = \theta_{FE}, E = \rho\delta / [1 - 1/(\epsilon\lambda)])$, where $\theta_{FE} = \delta (1 - \epsilon) / (\lambda\epsilon - 1)$. Since both curves rotate in opposite directions, the overall effect on $\theta$ is ambiguous.

The same ambiguity applies to the effect of faster learning on the growth rate. Nevertheless, the first order effect on the growth rate is positive, since faster learning increases the share of experienced incumbents. In Section 5 we will calibrate the model economy, and for reasonable parameter values we observe that an increase in the learning rate raises the research intensity and the growth rate in the exogenous learning model.

The effects of changes in the remaining parameters on research intensity are intuitive and are analogous to results of Grossman and Helpman (1991). For example, an increase in the effective labor force $L/\rho$, shifts the $(LL)$ curve up and the research intensity increases. Higher research intensity decreases the share of experienced, (new producers start with low experience), but total R&D efforts, $(\theta\mu_2)$, increases, and so does the growth rate. Because inexperienced incumbents produce more and have lower profits than experienced incumbents, total production increases and the profit rate declines. We can summarize the effects of changes in parameters on research intensity for the exogenous learning case as follows:

$$\begin{array}{lll} \partial\theta/\partial\alpha \gtrless 0, & \partial\theta/\partial L > 0, & \partial\theta/\partial\epsilon > 0, \\ \partial\theta/\partial\lambda > 0, & \partial\theta/\partial\rho < 0, & \partial\theta/\partial\delta \gtrless 0. \end{array}$$

### 4.2. Endogenous Learning-by-Doing: Qualitative Analysis

We now consider the two endogenous learning cases: for the first case (external LBD) the learning rate for a good depends on aggregate output of inexperienced producers, and for the second case (internal LBD) the learning rate for a good depends only on the output level of that good. We will focus on the analysis of internal learning, since external learning is essentially the same as exogenous learning. The endogeneity of the learning rate introduces a second round effect, whereby (endogenous) changes in $\alpha$ affect $\theta$. These secondary effects may amplify or dampen the effects relative to the model with exogenous learning.

Consider an increase in the size of the labor force $L$. With exogenous learning this will increase the research intensity $\theta$, the share of inexperienced incumbents $\mu_1$, and the expenditure level $E$. The last two effects will increase total production by inexperienced incumbents, and thereby raise the learning rate $\alpha$. The effect of higher $\alpha$ on $\theta$ is ambiguous, as we have seen in the previous section. Suppose that this effect is positive, so that faster learning raises the research intensity. Then, the second round effect of higher $L$ on $\theta$ through $\alpha$ reinforces the direct effect. The same analysis applies to changes in other parameters.

To evaluate the equilibrium responses under internal learning, assume that the limit price constraint is not binding, (i.e. an incumbent which maximizes profit selects a smaller quantity, and higher price, than is needed to exclude the follower from the market). Note that here, unlike the external learning case, an increase in research intensity *lowers* the rate of learning, because it reduces the capital gain from becoming experienced. This effect will dampen the response of the economy on a balanced growth path to parameter changes.

For the example of an increased labor force considered above, we have a positive first round effect of $L$ on $\theta$, as in the exogenous learning case. Inexperienced incumbents will now respond to the greater vulnerability of the profitable high-experience status by lowering their production levels, thereby reducing their own learning rate. Assuming a positive relation between the learning rate and research intensity, the reduction in the learning rate will eliminate part of the increase in the research intensity which was generated by the increase in $L$. Further results cannot be obtained through analytical methods only, and we proceed with a numerical analysis of a calibrated version of the economy.

## 5. Endogenous Learning in a Calibrated Model

We now study the quantitative implications of the interaction between LBD and R&D in an economy in which learning is endogenous. This analysis shows that the response of the learning rate to changes in the learning function and the labor endowment critically depends on whether producers take into account the effects of their production levels on the learning rate. In addition, the numerical analysis reveals that parameter values for the LBD technology also play a major role in determining the response of the growth rate to the exogenous changes considered. For example, we find that an increase of the labor endowment will raise the growth rate in an economy with exogenous or external learning, but will have the opposite effect in an economy with internal learning. Our study is based on a numerical analysis of the economy. We select parameter values such that the balanced growth path of the economy matches a selection of observations on average growth rates, interest rates, R&D and LBD activities.

### 5.1. Calibration

Our economy's balanced growth path is to replicate the growth experience of the post-war U.S. economy. We are therefore interested in balanced growth paths with an annual growth rate of two percent, $g = 0.02$, and an average interest rate of four percent, $r = 0.04$. The crucial part of our calibration exercise is the characterization of the R&D and the LBD process.

One can measure R&D competition through the rate at which the capital value of patents depreciates. Schankerman and Pakes (1984, 1986) estimate capital value depreciation rates using information on patent renewals. For a sample which includes the UK, Germany, France, the Netherlands and Switzerland from 1930-1939, Schankerman and Pakes (1984) estimate an annual depreciation rate for patent values of twenty five percent. For a post World War II sample, which includes three European countries, Schankerman and Pakes (1986) estimate annual depreciation rates between ten percent (Germany) and twenty percent (UK, France). In a related work, Caballero and Jaffe (1993) estimate the rate of decline in a firm's value which is attributable to a decline in its relative R&D position at an average of three and a half percent a year.[14] We interpret these depreciation rates in firms and patent values as the

[14] Their estimates range from 0% to 25% per year.

average rate at which the value of a fixed sample of experienced firms in our model declines over time, which is represented by the entry rate $\theta$. For our parameterization we assume that on the balanced growth path the capital value depreciation of an experienced incumbent is ten percent, $\theta = 0.1$, a value which is between the lower bound set by Caballero and Jaffe (1993) and the upper bound set by Schankerman and Pakes (1984, 1986).[15]

Our survey of empirical work on LBD in section 2 finds a wide range of estimates of the unit cost reductions and its precise dependence on cumulative output. Two categories of estimates can be identified in this literature, which we label by "narrow" and "wide" learning cases. The "narrow learning" case covers short term labor cost reductions for one product in a given production site. The empirical evidence on this case, which has been intensively studied, suggests a learning rate of about 20%, a potential cost reduction factor of 2, and a period of 3 to 5 years for achieving this efficiency gain. The "wide learning" case covers increased total factor productivity for a variety of similar products over the life cycle of a production site, and estimates of LBD benefits of this kind are more modest. For instance, Bahk and Gort (1993) find that a new plant is about eighty percent as productive as a mature plant, and that a plant attains maturity within six to ten years.

We believe that in order to assess the impact of learning on growth the "wide learning" case is more relevant than the "narrow learning" case. As noted most of the evidence on "narrow learning" refers to unit cost reductions for a product which is new for some production facility, but the product is not necessarily new for the economy. For example, under the "narrow learning" case we will observe that in a new pizza outlet the unit cost

[15] Note that $\theta$ is an endogenous variable in our model. The results reported here are the value of (exogenous) parameters needed so that in a steady state equilibrium the value of $\theta$ is 0.1.

of pizzas will initially decline. It is hard to see that the unit cost of production of a well-known product in a new production site should affect the economy's ability to develop new products. On the other hand it appears more reasonable that a higher general experience with production processes in plants with "wide learning" should increase the economy's ability to develop new products. We do, however, report the results for both parameter configurations, since the models respond rather differently to the changes we consider, depending on the configuration used.

We use these results from the empirical work on LBD to parametrize our economy as follows. First, we identify the experience component of quality obtained by learning in our model with the cost reductions due to learning in the empirical work on LBD. The scope of learning in our model is directly analogous to the potential efficiency gain attributed to LBD, yielding the alternative values for $\epsilon$ of 0.5, and 0.8 for the narrow and wide learning cases, respectively. We derive an estimate of the learning rate $\alpha$ by considering a collection of firms of measure one, all of them inexperienced. With each firm shifting stochastically to a high experience status at the rate $\alpha$, the average experience of these firms after some time $t$ has elapsed is:[16]

$$\bar{e}\left(t\right) = e^{-\alpha t} \cdot \epsilon + \left(1 - e^{-\alpha t}\right) \cdot 1$$

and the cumulative production of these firms up to time $T$ is:

$$\begin{aligned} Q(T) &= \int_0^T \left( e^{-\alpha\tau} \frac{E}{\epsilon\lambda} + (1 - e^{-\alpha\tau}) \frac{E}{\lambda} \right) d\tau \\ &= \frac{E}{\lambda} \left[ T + \left(1 - e^{-\alpha T}\right) \left(\frac{1}{\epsilon} - 1\right) / \alpha \right] \end{aligned}$$

[16]Note that we assume here that there is no R&D competition and experienced firms are not displaced by inexperienced firms.

For the narrow learning case, we plot the log of average experience against the log of cumulative production for a time span of one to three years. We then select the speed of learning $\alpha$ such that a straight line fitted to this curve based on OLS has slope 0.2, that is yields a learning rate of 20%. This procedure implies a speed of learning of $\alpha = 1$. In the wide learning case, for which there is no available link between output and efficiency gain, we simply assume that after ten years, average quality is within one percent of potential quality, $\bar{e}(10) = 0.99$, which yields $\alpha = 0.3$. To summarize, for the narrow learning case we have $\epsilon = 0.5$ and $\alpha = 1.0$, and for the wide learning case we have $\epsilon = 0.8$ and $\alpha = 0.3$.

The information provided above is sufficient to parametrize the model with exogenous learning. Given the R&D rate $\theta = 0.1$, and the learning rate $\alpha$ corresponding to each learning case, we determine the share of experienced firms on the balanced growth path $\mu_2 = \alpha / (\alpha + \theta)$. The improvement factor $\lambda$ is then determined through the growth rate of the economy $g = \mu_2 \theta \log \lambda$. We then use the free-entry condition and the labor market clearing condition to determine the values for the expenditure-labor force ratio $E/L$, and the R&D cost-labor force ratio $\rho/L$. We normalize the labor force at one.

Flow profits when inexperienced are lower under narrow learning than under the wide learning. This follows from the fact that the product improvement factor $\lambda$ turns out to be similar for both cases, while the initial experience level is much lower for the narrow learning case. In fact, initial experience in the narrow learning case is so low that a producer makes losses under those parameters as long as she remains inexperienced. Another implication of our parameterization is that in the wide learning case, about 12 percent of the labor force is employed in R&D activities, and that in about 25 percent of all industries the producer is inexperienced. For the narrow learning case, those shares are 7 and 9, respectively.

To compute the equilibrium when the learning rate is endogenous, (external and internal), we proceed as follows. For the case of external LBD we assume that the learning function takes the form:

$$\alpha = \alpha_0 X_1^{\alpha_1},$$

and we set $\alpha_1 = 0.5$, and $\alpha_0$ is determined from $\alpha$, which was separately derived above for each of the two learning cases. For the internal LBD case, we assume a learning function of the form:

$$\alpha = \max \{0, \alpha_0 x_1^{\alpha_1} - \alpha_2\}$$

where $\alpha_0, \alpha_1, \alpha_2 \geq 0$. This functional form assumes that a certain amount of production has to take place before learning actually occurs. Again we set $\alpha_1 = 0.5$. We want that inexperienced incumbents take advantage of the learning opportunities, and actually produce more than they would produce without learning. In particular we assume that the optimal production of an inexperienced incumbent is a multiple $\kappa$ of the output level implied by the limit price when inexperienced. We select $\kappa = 1.1$.

What will be important for our balanced growth path analysis is the elasticity of the learning function with respect to changes in production, $\eta = (\partial\alpha/\partial x_1)(x_1/\alpha)$. Simple algebra shows that (for both external and internal learning) this elasticity is:

$$\eta = \left[\frac{\kappa E/L}{\epsilon\lambda}\right] / \left[\alpha \left\{\frac{1-1/\lambda}{r+\theta}\frac{E}{L} - \frac{\rho}{L}\right\}\right].$$

Note that this elasticity has already been determined by the previous calibration procedure, and is independent of the parameter values of the learning function. The term in curly brackets represents the capital gain from becoming experienced. This capital gain is increasing in the scope of learning, that is lower values of $\epsilon$ imply lower elasticities of the learning function. For example, in the narrow learning case the elasticity is $\eta = 2.2$, and in the wide learning case it is $\eta = 9.9$.

We summarize the calibrated balanced growth paths for the two alternative parameter configurations in Table 1 below.

TABLE 1 - HERE

### 5.2. Response to Exogenous Changes

A partial list of the results from the numerical analysis are presented in Tables 2 and 3. These tables contain the elasticities of a subset of endogenous variables with respect to the labor endowment ($L$), the learning technology parameters, ($\alpha$ for exogenous learning, and $\alpha_0$ for the two endogenous learning cases), and the learning scope represented by the initial experience, ($\epsilon$). We list the effect of changes in these parameters on the growth rate $g$, the research intensity $\theta$, the share of experienced industries $\mu_2$, the average profit rate in manufacturing $\pi$, and the learning rate $\alpha$ when it is endogenous. Table 2 contains the results for the parametrization corresponding to the wide learning case, and Table 3 pertains to the narrow learning case.

The first conclusion to emerge from the calibrated model is that although there are some quantitative differences between the model with exogenous learning and the model with endogenous but external learning, the qualitative features are very similar. In both environments an increase in the labor endowment increases research intensity. The increase in research intensity offsets the decline in the share of experienced firms, and the growth rate, which is the product of the two, increases. These results are robust across the two parametric configurations, as can be seen by comparing the responses of $g$ to $L$ under exogenous and external learning in both Tables 2 and 3. Our results under exogenous and external LBD assumptions are also consistent with other works on LBD, which assume that firms do not internalize the LBD effects of their current production decision on future cost, (Stokey (1988), Young (1993), Lucas (1993) and Parente (1994)).

TABLE 2 HERE

In contrast to the above positive scale effects under exogenous and external LBD, the effects of an increase in the labor endowment when learning is internalized depend on the elasticity of the learning function. For the wide learning case with a high elasticity, an increase in the labor endowment actually *lowers* the growth rate in an economy with internal LBD, as can be seen in Table 2. This result obtains because the endogenous learning responds differently to a change in the research intensity, depending on whether producers' future efficiency is affected by their own production experience or by economy wide aggregates. With external LBD, the learning rate responds positively to the increased share of inexperienced firms following an increase in research intensity, and this almost completely offsets the initial reduction in the share of experienced firms. In contrast, when research intensity increases under internal LBD, there is nothing to offset the reduced capital value of being an experienced producer and learning becomes less attractive. Accordingly, producers respond by cutting down on (indirect) learning costs, (by reducing output), and the decline in the share of experienced firms is exacerbated, thus lowering the growth rate.

TABLE 3 HERE

The effects of changes in the speed of learning display similar sensitivity to the nature of the learning process. Under exogenous and external learning, an increase in the speed of learning increases both the research intensity and the share of experienced producers and thereby the growth rate. With internal learning the second round effects from the higher research intensity reduce the speed of learning and lower the share of experienced producers. Because the elasticity of the learning function is relatively high, the growth rate will decline, see Table 2.[17]

Lastly, a smaller learning scope, represented here by higher value of $\epsilon$, will have no impact on growth when learning is internal. This follows from our assumption that the output of inexperienced producers exceeds the level implied by limit pricing: in order to enhance their learning rate they produce so much that the price is even lower than the one required to just keep producers of previous generations of the good out of the market. Consequently, a lower quality gap between high and low experience goods, (which does not violate the non-binding limit price assumption), does not result in any change in the price differential between these goods, nor in any other endogenous variable.

In contrast, when learning is exogenous or external to producers, a higher quality of the low-experience good increases the capital value $V_1$, implying a higher R&D intensity, higher share of inexperienced producers, and higher learning rate in the external learning case, (despite the decrease in individual output by an inexperienced producer). Although the decrease in $\mu_2$ works against the increase in $\theta$ in terms of their growth implications, the overall impact on growth is positive.

[17] Parente (1994) in an economy with exogenous LBD, Young (1993) in an environment with external LBD, and Lucas (1993) in an environment where firms do not take into account the effect of their production decisions on learning, find that an improvement in the learning technology increases the growth rate.

## 6. Conclusion

Although we maintain the assumption of constant returns to scale in R&D, the model we have presented does not necessarily produce the positive “scale effects” of other R&D based growth models. In particular, we may have negative ”scale effects” in that the growth rate may decrease with the size of the economy. These results depend on the parameter values of the learning technology, and are more likely when learning is internal.

Under internal learning, factors that increase the intensity of R&D competition against individual producers might nevertheless reduce the growth rate in the economy. The potentially negative growth impact of intensified R&D works as follows: a more intense R&D competition against existing producers reduces their incentives to invest in LBD, the resulting lower average experience associated with slower LBD reduces the scope for R&D, thus reducing the growth rate. The first of these two effects merely reflects the capital theoretic nature of investment in LBD. In particular, it does not depend on the assumption that experience with existing products facilitates R&D for better ones, which drives the second effect. One could assume, instead, that R&D is equally likely to succeed with high as well as low experience products. However, this would remove any spillover effects from technological improvements. The assumption we have adopted seems like the easiest way to capture experience externalities in the development of new products.

The fact that low experience producers choose to reduce their output in the face of intensified R&D competition may also be interpreted as an attempt to prolong their low-experience status, in which they are immune to R&D competition under our assumptions. There is no easy way to disentangle this interpretation from the alternative one that we have provided, namely, that output is reduced because LBD becomes less profitable when R&D competition becomes more intense. In our computed equilibrium, producers choose output level well in excess of what is needed to capture 100% market share. This excess output is produced only to expedite the experience accumulating process. The implication that incentives to overproduce in this way diminish with the intensity of the R&D competition is likely to emerge even in models in which R&D intensity is independent of the share of experienced producers.

Modifying the specification of external learning, so that the learning rate depends on the production volume of experienced, (rather than inexperienced), producers does not affect our results in any quantitatively important way. For example following an increase in the labor endowment, the growth rate of the economy increases less when inexperienced firms learn from experienced firms. The intuition for this is fairly straightforward. An increase in the labor endowment raises R&D efforts, thereby lowering the share of experienced producers. This in turn tends to lower total production by experienced producers, thus reducing the learning rate and exacerbating the decline in the share of experienced producers. Hence the growth rate does not increase as much as when inexperienced firms learn from other inexperienced firms.

A modification that could conceivably overturn our results involves internalizing the beneficial R&D implications of LBD. If the spillover from experience with existing products to the development of new ones is internalized this way, and if an incumbent can change by his own R&D efforts his probability of winning the R&D race to the next basic quality, then the optimal incumbent's response to an intensified R&D competition might well be to increase, rather than decrease, his output. This kind of an inherent R&D incumbent's advantage considerably weakens free entry into R&D activities, and probably fits the nature of R&D competition in industries which are highly concentrated to begin with.

We interpret our results as showing that reasonable modifications of the "linear" R&D based endogenous growth model can overcome some of its empirical failings. What is demonstrated in this paper is the need to study learning processes more closely in order to fully understand their growth implications. At the micro level we need more reliable parameter values for describing the learning and R&D technologies. At the aggregate or industry-wide level, we have to measure the extent to which LBD benefits are external or internal.

## References


[1] Alchian, A. 1959. Costs and Output. In *The Allocation of Economic Resources: Essays in Honor of B. F. Haley*, ed. M. Abramowitz. Stanford CA: Stanford University Press.

[2] Alchian, A. 1963. Reliability of Progress Curves in Air-Frame Production. *Econometrica* 31, 679-693.

[3] Argote, L., S. L. Backman, and D. Epple. 1990. The Persistence and Transfer of Learning in Industrial Settings. *Management Science* 36, 140-154.

[4] Asher, H. 1956. Cost-Quantity Relationships in the Airframe Industry. Santa Monica, CA: RAND Corporation.

[5] Bahk, B.-H., and M. Gort. 1993. Decomposing Learning by Doing in New Plants. *Journal of Political Economy* 101: 561-583.

[6] Baloff, N. 1966. Startups in Machine-Intensive Production Systems. *Journal of Industrial Engineering* 17: 25-32.

[7] Baloff, N. 1971. Extension of the Learning Curve-Some Empirical Results. *Operations Research Quarterly* 22: 329-240.

[8] Billon, S. A. 1966. Industrial Learning Curves and Forecasting Production Requirements. *Management International Review* 6: 65-96.

[9] Caballero, R. J. and Jaffe, A. B. 1993. How High are the Giants' Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth. In *NBER Macroeconomics Annual 1993*, eds. O. J. Blanchard and S. Fischer, 15-74. Cambridge MA: MIT Press.

[10] Cole, R. 1958. Increasing Utilization of the Cost Quantity Relationship in Manufacturing. *Journal of Industrial Engineering* 9: 173-177.

[11] Conway, R., and A. Schultz. 1959. The Maufacturing Progress Function. *Journal of Industrial Engineering* 10: 39-53.

[12] Grossman, G., and E. Helpman. 1991. Quality Ladders in the Theory of Growth. *Review of Economic Studies* 58: 43-61.

[13] Hall, G., and S. Howell. 1985. The Experience Curve from the Economist's Perspective. *Strategic Management Journal* 6: 197-212.

[14] Hirsch, W. Z. 1952. Manufacturing Progress Functions. *Review of Economics and Statistics* 34: 143-155.

[15] Irwin, D. A. and Klenow, P. J. 1994. Learning-by-Doing Spillovers in the Semiconductor Industry. *Journal of Political Economy* 102: 1200-1227.

[16] Jovanovic, B. and G. MacDonald. 1994. Competitive Diffusion. *Journal of Political Economy* 102: 24-52.

[17] Klenow, P. J. 1994. New Product Innovations. Graduate School of Business. University of Chicago. Mimeo.

[18] Lucas, R. E. Jr. 1988. On the Mechanics of Economic Development. *Journal of Monetary Economics* 22: 3-42.

[19] Lucas, R. E. Jr. 1993. Making a Miracle. *Econometrica* 61: 251-272.

[20] Parente, S. 1994. Technology Adoption, Learning-by-Doing and Economic Growth. *Journal of Economic Theory* 63August): 346-369.

[21] Rapping, L. A. 1965. Learning and World War II Production Functions. *Review of Economics and Statistics* 47: 81-86.

[22] Romer, P. 1986. Increasing Returns and Long Run Growth. *Journal of Political Economy* 94(5): 1002-1037.

[23] Schankerman, M., and Pakes, A. 1984. The Rate of Obsolescence of Patents, Research Gestation Lags, and the Private Rate of Return to Research Resources. In *R&D, Patents, and Productivity*, ed. Zvi Griliches, 73-88. Chicago and London: University of Chicago Press.

[24] Schankerman, M., and Pakes, A. 1986. Estimates of the Value of Patent Rights in European Countries during the Post-1950 Period. *Economic Journal* 96: 1052-1076.

[25] Searle, A. D. 1945. Productivity Changes in Selected Wartime Shipbuilding Programs. *Monthly Labor Review* 61: 1132-1147.

[26] Sheshinski, E. 1967. Tests of the Learning by Doing Hypothesis. *Review of Economics and Statistics* 49: 568-578.

[27] Stokey, N. L. 1988. Learning by Doing and the Introduction of New Goods. *Journal of Political Economy* 96: 701-717.

[28] Wright, T. P. 1936. Factors Affecting the Cost of Airplanes. *Journal of Aeronautical Sciences* 3: 122-128.

[29] Young, A. 1993. Invention and Bounded Learning by Doing. *Journal of Political Economy* 101: 443-472.

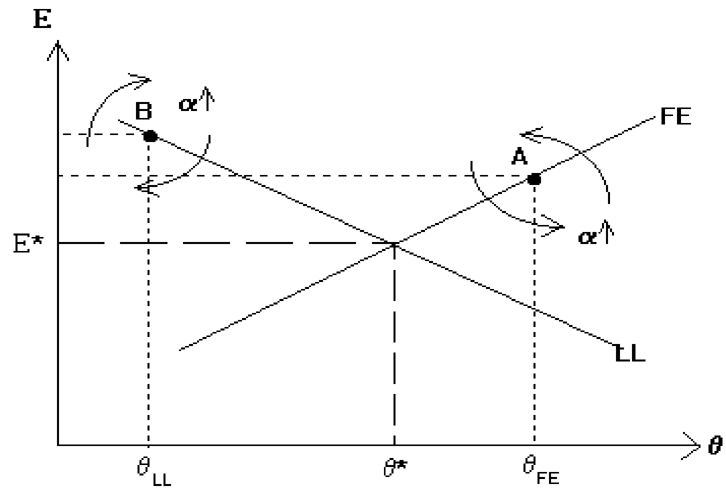


Figure 1. Equilibrium with Exogenous Learning-by-Doing

**Table 1: Calibration**

| | | |
|---|---|---|
| Discount Rate, | $\delta$ | 0.04 |
| Growth Rate, | $g$ | 0.02 |
| Research Intensity, | $\theta$ | 0.10 |
| | | |
| Case 1: Wide Learning | | |
| Basic Improvement Rate, | $\lambda$ | 1.31 |
| Initial Experience, | $\epsilon$ | 0.80 |
| Learning Intensity, | $\alpha$ | 0.30 |
| Measure of Experienced Incumbents, | $\mu_2$ | 0.75 |
| Share of R&D employment in total employment, | | 0.12 |
| Profit rate, | $\pi/E$ | 0.18 |
| | | |
| Case 2: Narrow Learning | | |
| Basic Improvement Rate, | $\lambda$ | 1.25 |
| Initial Experience, | $\epsilon$ | 0.50 |
| Learning Intensity, | $\alpha$ | 1.00 |
| Measure of Experienced Incumbents, | $\mu_2$ | 0.91 |
| Share of R&D employment in total employment, | | 0.07 |
| Profit rate, | $\pi/E$ | 0.12 |

**Table 2: Balanced Growth Path Analysis**

Wide Learning Case: $\epsilon = 0.8$ and $\alpha = 0.3$

Elasticity of variable $y$ with respect to changes in parameter $x$.

| Variable | Exogenous Learning | | | External Learning | | | Internal Learning | |
|---|---|---|---|---|---|---|---|---|
| $y$ | $\epsilon$ | $\alpha$ | $L$ | $\epsilon$ | $\alpha_0$ | $L$ | $\alpha_0$ | $L$ |
| $g$ | 1.89 | 0.11 | 1.02 | 1.98 | 0.22 | 1.27 | −54.24 | −29.76 |
| $\theta$ | 2.53 | 0.02 | 1.39 | 2.54 | 0.04 | 1.45 | 2.79 | 2.31 |
| $\mu_2$ | −0.63 | 0.09 | −0.37 | −0.56 | 0.17 | −0.18 | −57.01 | −31.94 |
| $\alpha$ | | | | 0.29 | 0.74 | 0.73 | −217.79 | −103.99 |
| $\pi$ | 0.80 | 0.07 | −0.29 | 0.85 | 0.13 | −0.14 | −36.89 | −21.08 |

**Table 3: Balanced Growth Path Analysis**

Narrow Learning Case: $\epsilon = 0.5$ and $\alpha = 1.0$

Elasticity of variable $y$ with respect to changes in parameter $x$.

| Variable | Exogenous Learning | | | External Learning | | | Internal Learning | |
|---|---|---|---|---|---|---|---|---|
| $y$ | $\epsilon$ | $\alpha$ | $L$ | $\epsilon$ | $\alpha_0$ | $L$ | $\alpha_0$ | $L$ |
| $g$ | 1.40 | 0.85 | 0.68 | 1.50 | 0.55 | 1.16 | 1.92 | 1.64 |
| $\theta$ | 1.54 | 0.83 | 0.75 | 1.63 | 0.52 | 1.20 | 3.13 | 1.94 |
| $\mu_2$ | −0.14 | 0.02 | −0.08 | −0.14 | 0.03 | −0.05 | −1.21 | −0.30 |
| $\alpha$ | | | | 0.14 | 0.84 | 0.70 | −10.15 | −2.81 |
| $\pi$ | 0.35 | 0.16 | −0.42 | 0.38 | 0.17 | −0.27 | −0.29 | −0.43 |