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Should Optimal Discretionary Monetary Policy Look at Money?*

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Abstract

This paper examines whether monetary indicators are useful in implementing optimal discretionary monetary policy when the policy maker has incomplete information about the environment. We find that money does not contain useful information for the policy maker, if we calibrate the model to the U.S. economy. If money demand were to be appreciably less variable, observations on money could be useful in response to productivity shocks but would be harmful in response to money demand shocks. We provide an incomplete information example where equilibrium welfare declines when the money demand volatility decreases.

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Keywords: monetary policy, sticky prices, optimal time-consistent policy, asymmetric incomplete information

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1. Introduction

Over the years we have participated in many discussions concerning appropriate monetary policy actions. While most central banks use an interest rate instrument in the pursuit of monetary policy, participants in these discussions frequently suggest that a key consideration for the setting of this instrument should be the behavior of money. One of the most prominent advocates of this approach, Friedman (1969), suggests that if money demand is well behaved, then monetary policy should respond to deviations of money growth from a preset target. Although there have been periods when the behavior of money has influenced the setting of short term interest rates, Friedman's prescription has not been followed in general. This is reflected in empirically estimated policy rules, such as Taylor (1993), which suggest that monetary authorities adjust their interest rate instrument in response to the behavior of inflation and some measure of real economic activity, but not in response to the behavior of money.

Is the current neglect of money in the pursuit of monetary policy justified? We reconsider this issue within the context of optimal monetary policy in an explicitly specified general equilibrium environment. In particular we study optimal time-consistent monetary policy in an economy where prices are sticky. If the policymaker has complete information about the state of the economy, optimal policy does respond to the state of the economy, but that state does not include the nominal money stock. Although monetary policy does not respond to the behavior of money, it may appear to do so to an outside observer, if that observer does not have complete information on the state of the economy, and the behavior of money reflects the behavior of the state. If the policymaker has incomplete information about the state of the economy, then optimal policy may respond to the behavior of money, if that behavior contains useful information for the policymaker about the underlying state of the economy.[1] For this to hold, money demand needs to be more stable than we observe for the U.S. economy. With incomplete information, however, it is not necessarily true that more information improves welfare. In particular, we provide an example where with more stable money demand the policymaker responds more aggressively to movements in money and thereby reduces economic welfare.

The plan of our paper is as follows. In section 2 we give a brief overview of monetary policy in the postwar United States and describe the extent to which the Federal Reserve System has used money to guide policy. We also investigate whether adding money to a Taylor-type rule would help explain Federal Reserve behavior. Only for the early to mid

[1] Recent work on optimal policy under incomplete information include Aoki (2000), Orphanides (1998), Smets(1998), Svensson and Woodford (2000, 2001), and Tetlow (2000).

1970s and the early 1980s do we find some evidence that the Federal Reserve raised interest rates in response to high money growth.

In section 3 we describe an economy with sticky prices due to staggered price setting in the spirit of Taylor (1980). In the model, real balances enter the representative agent's utility function, and we can derive a money demand function where velocity shocks represent preference shocks. Besides velocity shocks, we also consider productivity shocks. We parameterize the model based on the U.S. economy, and calculate a linear approximation to the optimal time-consistent monetary policy under full information. We characterize optimal monetary policy through impulse response functions and the behavior of estimated Taylor rules. We find that the optimal monetary policy responds to both shocks, but that the effects of productivity shocks dominate. In response to a positive productivity shock, optimal policy lowers interest rates. Taylor rules estimated in the model capture some but not all features of empirically estimated Taylor rules: while higher inflation is associated with higher interest rates, above average output is associated with lower interest rates. The last feature reflects the response to productivity shocks which are the major source of fluctuations in the model economy. Estimated policy rules also seem to indicate a desire for interest rate smoothing, although the underlying true policy rule has no role for this behavior. Finally, for some of the estimated policy rules, interest rates are negatively associated with money growth.

In section 4 we analyze our model when the private sector continues to have full information, but the monetary authority has incomplete information. In particular, we assume that the policymaker does not observe the state of the economy, but receives a noisy contemporaneous signal of the money stock and lagged noisy information about output. Our basic finding is that for a parameterization of the signal noise in money and the volatility of velocity, which is consistent with that observed in the United States, observing money in addition to output does not change the dynamics of the economy substantially. However, for appreciably less money demand volatility, information on money does improve the reponse to productivity shocks. This improved reponse to productivity shocks comes at a cost, in the sense that money demand shocks are now misconstrued as negative productivity shocks, and the policymaker's response to the signal can cause a recession. Overall, the economy with a more stable money demand is actually worse off in terms of unconditional expected utility. We conclude with a brief summary and some thoughts for future work. In the appendix we characterize time-consistent optimal policy for linear-quadratic control problems with incomplete and asymmetric information.

2. Evidence of the use of money in monetary policy

In this section we provide some evidence on the use of money in monetary policy. Our discussion concentrates on post-World War II Federal Reserve policy, but it is clear from work such as Bernanke and Mishkin (1992), Clarida and Gertler (1997), and Rich (1997) that other central banks occasionally do pay attention to the behavior of some monetary aggregate. We investigate the use of money in two ways. First, we ask if the federal funds rate was adjusted in response to deviations of actual money growth from an explicitly stated money growth target. Second, we ask if the inclusion of past money growth enters significantly into Taylor-rule type estimates of monetary policy.

2.1. Descriptive evidence

There is evidence that U.S. monetary policy responded to the behavior of M1 during the first half of the seventies and the first half of the eighties. For the first period, Hetzel (1981) describes how beginning in September 1972 an M1 target was specified in terms of a two-quarter growth rate. Target is somewhat of a misnomer because the Fed did not conduct monetary policy with the sole intention of hitting some fixed growth rate. However, the behavior of M1 did influence the setting of the funds rate at FOMC meetings and served as a device for indicating how the open market desk should vary the funds rate between meetings. For example, during the joint intervals October 1972 to August 1974, April 1975 to October 1975, and February 1977 to September 1979, the projected growth of M1 at the prevailing funds rate was above the midpoint of its tolerance range at 44 meetings and below the midpoint only 4 times. As a result, the FOMC raised the funds rate at 37 of these meetings and lowered the rate only seven times. Conversely, over the intervals September 1974 to March 1975 and November 1975 to January 1977, projected M1 growth was below target 14 times and above it on only three occasions. Over this period, the FOMC lowered the funds rate 13 times and raised it only four times. In particular, from the spring of 1973 to the fall of 1974, one could argue that the behavior of M1 constrained monetary policy in the sense that the Fed successfully hit its money growth targets over that period.

Although M1 targets were de-emphasized in 1982, there is still evidence that M1's behavior influenced policy over the period 1983-85 (see Dotsey [1996])[2]. During this period the ordering of variables in the FOMC's directive to the open market desk continually changed. The February, March, and May 1983 directives emphasized the behavior of money. As the year progressed, business activity and inflation became increasingly important, but in 1984

[2] There also exists a debate whether M1's behavior affected policy during the 1980-81 period. For differing views see Broaddus and Goodfriend (1984) and Bernanke and Mishkin (1992).

the growth rate of money was emphasized again. As documented in Goodfriend (1993), the Fed was faced with an inflation scare and as a result policy tightened. The Fed's focus on strong growth in both M1 and M2 no doubt provided useful political cover for tighter policy. But with the containment of inflation, emphasis on money waned and by 1985 there is no mention of it in the directive. Unfortunately, by mid-1985 the ordering of variables in the directive no longer changes and thus provides no information concerning the importance of money's behavior for policy. Further, M1 targets were formally abandoned in February of 1987.

2.2. Statistical evidence

To further investigate whether the behavior of money played any role in policy, we look at a Taylor-type reaction function augmented by the growth rate of M1, Taylor (1993) . Specifically we run the following regression using quarterly data,

$$R_t = a_0 + a_1 gap_t + a_2 (\log P_t - \log P_{t-4}) + a_3 R_{t-1} + a_4 (\log M1_t - \log M1_{t-4}) + e_t,$$

where R is the federal funds rate, *gap* is the output gap defined as the deviation of the log output from a quadratic trend where the estimated trend only uses information available during the relevant period. Thus, the gap is reestimated at each date. P_t is the consumer price index, and $M1$ is the money stock. The regression uses 40-quarter rolling windows and the estimated values of the coefficients together with their two-standard-deviation-confidence bands are depicted in Figure 1.[3] We use rolling windows because of the well-known time varying behavior of monetary policy. The coefficient on M1 growth is positive and takes on its largest value from the early to mid 1970's and in the mid 1980's, which is consistent with the descriptive evidence presented in the previous section. However, the coefficient is only statistically significant at the 5 percent level for the earlier sub-period, but is significant at the 10 percent level in the latter sub-period. Furthermore, starting in the late 1980s the confidence bands not only include zero, but the absolute value of the coefficient on money growth declines. As noted, this was when the Federal Reserve formally dropped M1 targets.

3. Optimal monetary policy with full information

We have seen that monetary policy occasionally responds to the behavior of money, but not always. We now ask whether monetary policy should respond to the behavior of money and, if

[3]The main message of the results are not appreciably different if we replace the output gap with growth in the output gap or output growth, or if we extend our rolling windows to 60 quarters.

yes, under what circumstances. To answer this question we study optimal monetary policy in an explicit dynamic general equilibrium framework. This approach has the advantage that the objective of the policymaker is well-defined, namely the welfare of the agents in the economy. We choose to look at time-consistent or discretionary monetary policy because we feel it may be a better representation of actual policy than full commitment, although both are extreme cases. At least in the United States, policy is decided as a sequence of individual policy actions rather than adherence to a well-defined rule. Indeed much academic policy advice suggests that a rule-like behavior be adopted. It is, however, true that current policy actions are somewhat constrained by long-run inflation objectives, but concerns about future inflation also enter into the discretionary policymaker's decision through his value function. He, however, takes policy decisions with respect to future inflation as outside his control. Therefore, policy issues that concern reputation and credibility are outside the scope of our modeling strategy. Given the experience with inflation scares and the variability of monetary policy over the last 30 years, neither the time-consistent nor the full commitment approach seems adequate. We feel there is something to be learned from both approaches.

3.1. The model

We wish to develop a model where the behavior of money is potentially important in the design of optimal policy. We do this by incorporating two important channels through which money may influence policy. First, changes in the demand for money arise from changes in preference parameters. In this setting, a monetary authority concerned with maximizing welfare may wish to react to changes in the demand for money. Second, in a world of incomplete information, money may convey useful information about state variables in the monetary authority's reaction function. In this section we abstract from the second channel and study the case of full information.

Our basic model includes an infinitely lived representative household with preferences over consumption, leisure, and real balances. The consumption good is produced with a large number of differentiated intermediate goods. Each intermediate good is produced by a monopolistically competitive firm with labor as the single input. Each intermediate goods firm sets a nominal price for its product, and this price is fixed for a finite number of periods. In particular, an equal number of firms can change their price each period. This type of staggered time-dependent pricing behavior, referred to as a Taylor contract, is a common methodology for introducing price stickiness into an otherwise neoclassical model.

3.1.1. The household

The representative household's utility is a function of consumption c_t, real money balances m_t, and the fraction of time spent working n_t,

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \log[\theta c_t^{\rho} + (1-\theta)\, z_{mt}^{1-\rho} m_t^{\rho}]^{1/\rho} + \chi \frac{(1-n_t)^{1-\phi} - 1}{1-\phi} \right\} \right], \tag{3.1}$$

where $\chi, \phi \geq 0$, $\rho \leq 1$, $0 < \beta < 1$, and z_{mt} is a preference shock. The notation E_t is used to denote the expectations conditional on the information available to the household at time t. The household's period budget constraint is

$$P_t c_t + B_{t+1} + M_{t+1} \leq W_t n_t + R_{t-1} B_t + M_t + \Pi_t, \tag{3.2}$$

where P_t (W_t) is the money price of consumption (labor), B_{t+1} (M_{t+1}) are the end-of-period holdings of nominal bonds (money), and R_{t-1} is the gross nominal interest rate on bonds.[4] The agent owns all firms in the economy, and Π_t is profit income from firms. In the following we will use the term *real* to denote nominal variables deflated by the price of consumption goods, and we use lower case letters to denote real variables. In particular, real balances are deflated end-of-period nominal balances $m_t = M_{t+1}/P_t$.

The first order conditions of the representative household's problem can be written as

$$\lambda_t w_t = \chi (1-n_t)^{-\phi}, \tag{3.3}$$

$$\lambda_t = \beta R_t E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}} \right], \tag{3.4}$$

$$m_t = z_{mt} c_t \left(\frac{1-\theta}{\theta} \frac{R_t}{R_t - 1} \right)^{1/(1-\rho)}, \tag{3.5}$$

$$\lambda_t = \Lambda(R_t, z_{mt}) / c_t, \tag{3.6}$$

$$\Lambda(R_t, z_{mt}) \equiv 1 / \left\{ 1 + \left(\frac{1-\theta}{\theta} \right)^{1/(1-\rho)} \left(\frac{R_t}{R_t - 1} \right)^{\rho/(1-\rho)} z_{mt} \right\}.$$

Equation (3.3) states that the marginal utility of leisure equals the real wage weighted by the marginal utility of consumption. Everything else unchanged, the consumer will work more with higher wages. Equation (3.4) describes the optimal savings behavior of individuals. If the return to saving rises, then households will consume less today, save more, and consume more in the future. Equation (3.5) is a money demand relationship, where real money demand depends on consumption and the opportunity cost of holding money. Equation (3.6) defines the Lagrange multiplier on the resource constraint, that is the marginal value of consumption.

[4] In an equilibrium, bonds are in zero net supply.

3.1.2. Firms

The consumption good is the final output of a constant returns to scale technology, which uses a continuum of differentiated intermediate goods as inputs, indexed $j \in [0,1]$. Total output as a function of intermediate goods $y(j)$ used is $c = \left[\int_0^1 y(j)^{(\varepsilon-1)/\varepsilon} dj\right]^{\varepsilon/(\varepsilon-1)}$, where $\varepsilon > 1$. Producers of the final good behave competitively in their markets, and given prices $P(j)$ for the intermediate goods, the nominal unit cost and price of the final good is

$$P = \left[\int_0^1 P(j)^{1-\varepsilon} dj\right]^{1/(1-\varepsilon)}. \tag{3.7}$$

For a given level of production, the cost-minimizing demand for an intermediate good j is

$$y(j) = [P(j)/P]^{-\varepsilon} c. \tag{3.8}$$

Each intermediate good is produced by a single firm, and j indexes both the firm and the good. Output of firm j is a function of labor $n(j)$ only,

$$y_t(j) = z_{yt} n_t(j), \tag{3.9}$$

where z_y is an aggregate technology shock. Each firm behaves competitively in the labor market, and takes wages as given. Real marginal cost in terms of final goods is then $\psi_t = w_t/z_{yt}$. Alternatively, the average mark-up in the economy is $1/\psi_t$. Since each intermediate good is unique, intermediate goods producers have some monopoly power, and they face downward sloping demand curves (3.8).

A firm is allowed to adjust its nominal price once every J periods, and it chooses a price that will maximize the expected value of the discounted stream of profits over that period. Since all intermediate goods producers are identical except for the time when they can adjust their price, we consider only symmetric equilibria where producers differ only according to how much time has elapsed since they last changed their price. Let $p_{\tau,t} = P^*_{t-\tau}/P_t$ denote the time t relative price of a firm which has set its price τ periods ago, $P^*_{t-\tau}$. These relative prices change with the price level

$$p_{\tau+1,t+1} = \frac{p_{\tau,t}}{P_{t+1}/P_t} \text{ for } \tau = 0, \ldots, J-2. \tag{3.10}$$

Given the sequence of relative prices the expected present value of intermediate goods producers can be defined recursively as[5]

$$\begin{aligned} v_{0,t} &= \max_{p_{0,t}} \left\{\pi_t(p_{0,t}) + E_t\left[\Delta_{t,t+1} v_{1,t+1}(p_{1,t+1})\right]\right\} \\ v_{\tau,t}(p_{\tau,t}) &= \pi_t(p_{\tau,t}) + E_t\left[\Delta_{t,t+1} v_{\tau+1,t+1}(p_{\tau+1,t+1})\right] \text{ for } \tau = 1, \ldots, J-2 \\ v_{J-1}(p_{J-1,t}) &= \pi_t(p_{J-1,t}) + E_t\left[\Delta_{t,t+1} v_{0,t+1}\right] \text{ for } \tau = J-1, \end{aligned}$$

[5] We have suppressed the dependence of the firm's value on variables other than it's own relative prices.

where real profits are $\pi_t(p_{\tau,t}) = (p_{\tau,t} - \psi_t)\, p_{t,\tau}^{-\varepsilon} c_t$, and $\Delta_{t,t+\tau} = \beta^\tau \lambda_{t+\tau}/\lambda_t$ is the discount factor according to which the representative household evaluates future consumption relative to current consumption. Assuming that the value functions are differentiable, we get a recursive definition of the marginal values of intermediate goods producers

$$\begin{aligned} 0 &= \pi_t'(p_{0,t}) + E_t\left[\Delta_{t,t+1} v_{1,t+1}'(p_{1,t+1})(P_t/P_{t+1})\right] \\ v_{\tau,t}'(p_{\tau,t}) &= \pi_t'(p_{\tau,t}) + E_t\left[\Delta_{t,t+1} v_{\tau+1,t+1}'(p_{\tau+1,t+1})(P_t/P_{t+1})\right] \text{ for } \tau = 1, \ldots, J-2 \\ v_{J-1,t}'(p_{J-1,t}) &= \pi_t'(p_{J-1,t}). \end{aligned} \tag{3.11}$$

Repeated substitution for the marginal value of a firm with preset prices yields the following representation of the profit-maximizing relative price choice

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{\tau=0}^{J-1} E_t\left[\Delta_{t,t+\tau}\psi_{t+\tau}(P_{t+\tau}/P_t)^{\varepsilon} y_{t+\tau}\right]}{\sum_{\tau=0}^{J-1} E_t\left[\Delta_{t,t+\tau}(P_{t+\tau}/P_t)^{\varepsilon-1} y_{t+\tau}\right]}. \tag{3.12}$$

If there is zero inflation and marginal cost is constant, the optimal relative price is a constant markup over marginal cost, $\mu = \varepsilon/(\varepsilon - 1)$. In general, however, a firm's pricing decision depends on future marginal costs, the future aggregate price level, future aggregate demand, and future discount rates. For example, if a firm expects marginal costs to rise in the future, or if it expects higher rates of inflation, it will choose a relatively higher current price for its product.

We obtain an "aggregate" production function from the demand function (3.8) for intermediate goods, the production function of intermediate goods (3.9), and labor market clearing

$$y_t = z_{yt} a_t n_t \text{ with } a_t = \left[\frac{1}{J}\sum_{\tau=0}^{J-1} p_{\tau,t}^{-\varepsilon}\right]^{-1} \tag{3.13}$$

Production efficiency requires equal use of intermediate inputs, given that the final goods production function is symmetric and concave with respect to intermediate inputs. Yet, in an economy with sticky prices and inflation, different intermediate goods producers charge different prices and therefore sell different quantities. Production in the economy with sticky prices is then in general inefficient, and the allocational efficiency coefficient $a_t \leq 1$ reflects the distortion introduced by unequal relative prices. Finally, these relative prices have to satisfy the following adding-up constraint, derived from the aggregate price index (3.7),

$$1 = \frac{1}{J}\sum_{\tau=0}^{J-1} p_{\tau,t}^{1-\varepsilon}. \tag{3.14}$$

To complete the model, we assume that aggregate productivity and the money demand preference shifter follow stationary AR(1) processes

$$\begin{aligned} \ln z_{yt} &= \gamma_y \ln z_{y,t-1} + u_{yt} \\ \ln z_{mt} &= \gamma_m \ln z_{m,t-1} + u_{mt} \end{aligned} \tag{3.15}$$

with $|\gamma_i| < 1$, $E[u_{it}] = 0$, $E[u_{it}^2] = \sigma_i^2$, for $i = y, m$.

3.2. Optimal time-consistent monetary policy

The policymaker follows a policy which maximizes the expected present value of the representative agent's lifetime utility subject to the restriction that the allocation can be supported as a competitive equilibrium. The agent's current period utility $u(x, y, R)$ is a function of the state of the economy x, other non-predetermined variables y, and the policy instrument, which we take to be the nominal interest rate R. We assume that the policymaker cannot commit to future policy actions, and for this reason we study Markov-perfect equilibria. In a Markov-perfect equilibrium we can view policy as being determined by a sequence of independent policymakers, and today's policymaker assumes that future policymakers will select the policy instrument as a given function of the state, $R_s = F_s(x_s)$ for $s > t$. Also, given the decision rules of future policymakers, next period's non-predetermined variables and the lifetime utility of the representative agent from period $t + 1$ on will be given by the functions $G_{t+1}(x_{t+1})$ and $V_{t+1}(x_{t+1})$. We represent the competitive equilibrium restrictions through a system of equations that represent the law of motion for state variables C_x, and restrictions on non-predetermined variables derived from market-clearing and optimizing behavior C_y. Given these restrictions, the policymaker chooses the nominal interest rate and non-predetermined variables optimally[6]

$$\begin{aligned} V_t(x_t) = \quad & \max_{R_t, y_t, x_{t+1}} u(x_t, y_t, R_t) + \beta E_t[V_{t+1}(x_{t+1})] \\ & \text{s.t. } x_{t+1} = C_x(x_t, y_t, R_t, u_{x,t+1}) \\ & \quad E_t\left[C_{y1}(x_{t+1}, y_{t+1}, R_{t+1})\right] = C_{y0}(x_t, y_t, R_t) \\ & \quad y_{t+1} = G_{t+1}(x_{t+1}) \text{ and } R_{t+1} = F_{t+1}(x_{t+1}). \end{aligned} \tag{3.16}$$

The utility maximizing choice implies policy functions for today's instrument and non-predetermined variables, $R_t = F_t(x_t)$ and $y_t = G_t(x_t)$, and a value function $V_t(x_t)$ which reflects maximal lifetime utility of the representative agent from today on. A stationary Markov-perfect equilibrium for this problem is characterized by the triple $Z = (F, G, V)$ such that (3.16) maps Z into itself.

[6] When the policy maker has full information, the distinction between policy instruments and non-predetermined variables has no substantive content.

We study a linear-quadratic approximation of the model, and a full description of our methodology can be found in the technical appendix of Dotsey and Hornstein (2001). Briefly, conditional on an initial guess of the steady state nominal interest rate, we can solve for the steady state of the competitive equilibrium. We then construct a linear-quadratic approximation of the objective function and a linear approximation of the competitive equilibrium constraints around this steady state. We solve for the Markov-perfect equilibrium of the linear-quadratic approximation following Svensson and Woodford (2000) and obtain an optimal policy $R_t = Fx_t$ and $y_t = Gx_t$. We use the optimal policy together with the linearized law of motion for the state variables to determine the steady state of the approximation, which includes the steady state of the nominal interest rate. We adjust the initial guess of the steady state nominal interest rate until the two rates are the same.

3.2.1. The representation of our specific problem

In a standard rational expectations equilibrium, when the policy rule specifies the choice of policy instrument R_t as some given function of state and flow variables, we treat the lagged relative prices $p_{\tau,t-1}$ for $\tau = 0, \ldots, J-2$, which have been set by firms in the past $J-1$ periods as state variables. From the point of view of the planning problem, however, nominal levels are of no concern. The equations that characterize the competitive equilibrium in any given period involve real variables, relative prices, the nominal interest rate, and the inflation rate, but not the current-period price level. Given that the price level is arbitrary, past nominal prices impose restrictions on the current allocation only through their relative prices. To clarify this point define the normalized lagged prices as $q_{\tau,t} = p_{\tau-1,t-1}/p_{J-2,t-1}$, for $\tau = 1, \ldots, J-2$. Using the transition equation for relative prices (3.10), we can rewrite the constraint on relative prices (3.14) as

$$1 = \frac{1}{J}\left[p_{0,t}^{1-\varepsilon} + \left(\sum_{\tau=1}^{J-1} {q_{\tau,t}}^{1-\varepsilon}\right)\left(p_{J-2,t-1}\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon}\right]$$

with $q_{J-1,t} \equiv 1$. Since the policymaker is free to choose the current inflation rate, the level of lagged relative prices, that is $p_{J-2,t-1}$ does not represent a restriction on the policymaker's choices; it is not pay-off relevant. Only normalized lagged prices constrain the policymaker's choices and should therefore be included as state variables in a Markov-perfect equilibrium. Finally, the normalized lagged prices evolve according to

$$q_{\tau,t+1} = \frac{p_{\tau-1,t}}{p_{J-2,t}} \text{ for } \tau = 1, \ldots, J-2. \tag{3.17}$$

We use the equations that define the competitive equilibrium, (3.3), (3.4), (3.5), (3.6), (3.11), (3.13), (3.14), (3.15), and (3.17) to define the dynamic constraints of the planning problem. The state variables are the normalized lagged prices and the exogenous shocks, $x_t = [q_{1t}, \ldots, q_{J-2,t}, z_{yt}, z_{mt}]$. A convenient choice of the flow variables includes consumption, the relative price of the current price adjusting firm, and the marginal value of firms that have changed their prices in previous periods, $y_t = [c_t, p_{0t}, v'_{1t}, \ldots, v'_{J-2,t}]$. Solving for the behavior of these variables allows us to recover the behavior of all the other variables in the model.

In order to perform our numerical analysis we have to parameterize the model economy. Table 1 lists the parameter values and implied steady state values. We choose the time preference parameter β such that the annual real rate of interest is 4 percent. We select the leisure parameters χ and ϕ such that agents work approximately 25 percent of total hours and the implied labor supply elasticity is slightly greater than one, which is consistent with estimates in Mulligan (1998). We choose ρ so that the interest elasticity of money demand is -0.1. This elasticity is within the bounds of most empirical estimates for M1. Likewise we choose θ to make the velocity of money equal to 1.12, which is the average value of M1 velocity over the period 1959-99. Finally, consistent with the work of Basu and Fernald (1997), ϵ is calibrated to yield a markup of 10 percent. The autocorrelation coefficients and variances for the technology shock are roughly consistent with the values used in the literature on quantitative dynamic general equilibrium models. The process for the money demand shock is derived from an M1 demand function estimated for the United States from 1970 to 1999. Thus our parameterization is broadly consistent with both the literature and U.S. data.

3.3. The optimal time-consistent policy function

We now characterize our approximation of the Markov-perfect equilibrium. We have outlined our solution procedure in section 3.2. We first describe the steady state nominal interest rate, and then interpret the behavior of the nominal interest rate off the steady state. The two parts are interrelated since in Markov-perfect equilibria the steady state depends on off-steady state policy decisions.

Conditional on the parametrization of our economy we find that the steady state nominal interest rate is 3.52 percent on a quarterly basis. Given the 1 percent real interest rate, this implies an annual inflation rate of about 10 percent. This number is large relative to current inflation rates in most OECD countries, although many countries have experienced inflation rates of that magnitude over the last 30 years. The steady state inflation rate is also high relative to what would occur under a policy of full commitment.

The high optimal inflation in this model occurs for the following reasons. In our economy, the policymaker faces several distortions. There is the standard feature that real balances are too low unless the nominal net-interest rate equals zero. There are also two other distortions that tend to become more important in an economy where the policymaker cannot commit to future policy choices. First, monopolisticaly competitive firms set their price as a markup over marginal cost, and production is inefficiently low. This distortion creates a desire to inflate, which lowers the average markup because not all firms can adjust their price. Second, with inflation and staggered price setting, firms differ according to the prices they charge, and the quantities they produce and sell. Because the different firms' intermediate goods enter final goods production symmetrically, this dispersion in the production allocation is inefficient. The optimal response to this distortion is to lower the inflation rate, which reduces the price and output dispersion across intermediate goods-producing firms.[7]

For a policymaker who cannot commit to future actions, the relative importance of the markup and relative price distortion changes directly with the number of periods for which prices are fixed. Loosely speaking, if prices are preset for a longer duration, then the impact of contemporaneous inflation on the current average markup increases, and the impact on the current relative price distortion decreases. This happens because there are relatively more firms with preset prices, and the fraction of firms whose relative price can be affected declines. Since the policymaker cannot commit to future actions, he tends to focus on the contemporaneous impact of his actions and discounts the impact on future average markups and relative price distortions.

In experiments, not reported here, we have found that the steady state inflation rate is reduced by one-half if $J = 2$, and that the steady state inflation rate is close to the Friedman rule if there is full commitment. Thus our solution leads to two observations. One is that monetary authorities have done better than the discretionary equilibrium, perhaps because they have access to some form of commitment technology. The other is that time-dependent pricing may not be an accurate depiction of the way firms behave. We conjecture that a state-dependent pricing mechanism would result in greater price flexibility and less incentive for the monetary authority to inflate.

Optimal policy lowers the interest rate in response to higher productivity and increases the interest rate in response to higher money demand. The response of the interest rate to percentage deviations of the state variables from their steady state values is[8]

$$\hat{R}_t = 0.22\hat{q}_{1t} + 0.019\hat{q}_{2t} - 0.13\hat{z}_{yt} + 0.0058\hat{z}_{mt}.$$

We have found Figure 2 useful for interpreting how policy responds to deviations of

[7] For a detailed discussion see Khan, King, and Wolman (2000).

[8] Hats denote percentage deviations from steady state values.

state variables from their steady state. Figure 2 displays the inital relative price p_{0t} and marginal cost ψ_t, conditional on the contemporaneous policy and state variables R_t, $q_{1t}, \ldots,$ $q_{J-2,t}$, z_{yt}, z_{mt}, and next period's variables λ_{t+1}, $p_{1,t+1}$, $v'_{1,t+1}$. The initial relative price and marginal cost reflect the two distortions the policymaker tries to affect: a higher marginal cost is equivalent to a lower average markup $1/\psi_t$, and a higher initial relative price implies a lower allocational effciency a_t.[9] The curve labeled LL is based on the household's optimality conditions (3.3), (3.4), and (3.6), and market clearing conditions (3.13), (3.14). The curve labeled PP is based on the optimal price-setting equation (3.11). For our discussion we assume that the utility function is logarithmic in leisure, $\phi = 1$, and the two curves are given by

$$\begin{aligned} c_t &= p_{0t} \frac{\Lambda\left(R_t, z_{mt}\right)}{R_t} \frac{1}{\beta \lambda_{t+1} p_{1,t+1}}, \\ \psi_t &= \chi / \left\{ \Lambda\left(R_t, z_{mt}\right) \left(z_{yt}/c_t - 1/a_t\right) \right\}, && \text{(LL)} \\ p_{0t}^{1-\varepsilon} \left[(\varepsilon - 1) - \varepsilon \psi_t p_{0t}^{-1} \right] &= \beta \lambda_{t+1} v'_{1,t+1} p_{1,t+1} / \Lambda\left(R_t, z_{mt}\right). && \text{(PP)} \end{aligned}$$

The intersection of the two curves represents a "temporary" equilibrium in which both the firm's and the agent's first order conditions are satisfied. It is a "temporary" equilibrium, since it is conditional on given values of future variables that may depend on current choices.

Both curves are upward sloping, but numerical analysis shows that at the steady state the LL curve is steeper than the PP curve. Regarding the LL curve, a higher p_0 implies a higher inflation rate $P_{t+1}/P_t = p_{0t}/p_{1,t+1}$, that is a lower real rate, and therefore higher consumption today. Labor demand increases because production increases, and due to the higher p_0 production is less efficient (a declines). Overall the real wage and therefore marginal cost has to increase along LL. The left-hand side of equation (PP) is proportional to the negative of marginal profit of a price-adjusting firm, and because the steady state marginal profit of such a firm is negative and period profits are concave in p_0, the direct effect of an increase in p_0 is positive. For given values of future variables, marginal cost has to increase to maintain the equilibrium.

The policymaker faces a trade-off between the markup and the relative price distortion. A higher nominal interest rate increases the real interest rate, consumption declines, and the LL curve shifts down. A higher nominal interest rate also shifts down the PP curve, since Λ

[9] From equations (3.13) and (3.14), the elasticity of the relative price distortion with respect to the relative price is

$$\frac{\partial a^{-1}}{\partial p_0} \frac{p_0}{a^{-1}} = \epsilon \frac{p_0^{1-\epsilon}/J}{1 - p_0^{1-\epsilon}/J} \frac{p_0 a^{-1} - 1}{p_0 a^{-1}} > 0.$$

Notice that the magnitude of the derivative declines with the number of periods J over which prices cannot be adjusted.

is decreasing in the nominal interest rate. For our parametrization, the effect of a nominal interest rate change on the PP curve is small relative to the effect on the LL curve, and we will ignore the effect on the PP curve. Given the positive slope of the PP curve, marginal cost and the initial relative price increase.

We interpret the response of optimal policy to deviations of state variables from their steady state as an attempt of the policymaker to return the economy to a desired combination of markup and relative price distortion. Consider first an increase in the normalized lagged price q_j for $j = 1, 2$. Near the steady state the relative price distortion is increasing in q_j, that is allocational efficiency a declines. The decline in a reduces the denominator of the term on the right hand side of (LL) and marginal cost must rise for any given value of p_0. Therefore, the LL curve shifts up. This shift reduces the marginal cost ψ and the initial relative price p_0, that is it increases the average markup and and increases allocational efficiency. In order to offset the impact on the distortions, the policymaker increases the nominal interest rate and reverses the shift of the LL curve.

Notice that as the number of periods increases for which a firm's price is preset, the policymaker's impact on the distribution of relative prices declines, because much of the distribution is inherited, cf footnote 99. In terms of Figure 2 this means that the impact of the initial relative price p_0 on the relative price distortion becomes smaller, and the (LL) curve becomes less steep. Therefore, a nominal rate increase has a bigger effect on the average markup compared to the relative price distortion, and the monetary authority on average chooses a higher nominal interest rate and inflation rate.

Now consider the interest rate response to a productivity increase. Higher productivity shifts the LL curve down, which in equilibrium lowers the mark-up distortion and increases the relative price distortion. In order to return the economy towards the original combination of distortions, the interest rate is lowered and the LL curve shifts back towards its original position. In response to a money demand increase, the PP curve shifts down very slightly implying a higher markup and lower p_0. In order to move back in the direction of the steady state trade-off between the two distortions, the LL curve must also shift down. This shift is accomplished by an increase in the nominal interest rate. In each case the interest rate response is to return the economy toward its steady state equilibrium.

3.4. The behavior of the economy with full information

The preceding discussion gives some intuition on how a policymaker will respond to various shocks. The discussion, however, holds fixed the values of future variables, and it does not consider the dynamic interaction of private sector decisions and policy decisions. We now describe the qualitative and quantiative dynamic features using impulse response functions. We also discuss whether our model generates reduced form policy rules which are similar to Taylor-rules estimated for the U.S. economy.

3.4.1. Impulse response functions

We display the impulse reponse functions for a productivity shock (panel A) and a money demand shock (panel B) in Figure 3. Following a productivity shock the policymaker lowers the nominal interest rate, and the economy's behavior is remarkably close to that of an economy without frictions. In an economy with flexible prices the income and substitution effects of higher real wages cancel, and employment does not move. The output response then just mirrors the time path for productivity. In an economy with sticky prices, optimal monetary policy with commitment almost perfectly replicates the outcome for the economy without frictions, Khan, King, and Wollman (2000). As we can see here, optimal policy without commitment appears to be a bit more opportunistic in the sense that it generates a small but noticable increase of employment, and therfore a bigger output response. In lowering the nominal interest rate, the policymaker is able to reduce the markup, as is evident from the small increase in marginal costs, and simultaneously achieves an increase in efficiency, as indicated by the decline of the initial relative price. The increase in output, as well as the decline in the nominal interest rate, increases real balances. One period after the shock, adjusting firms also lower their price as do the remaining firms when their turn for price adjustment occurs.

The effects of a money demand shock are quantitatively small, relative to the effects of a productivity shock. The only exception are real balances.[10] For output movements, a comparison of panels A and B in Figure 3 shows that productivity shocks are much more important than are money demand shocks. Thus without observations on the underlying shocks, one is more likely to attribute a given increase in output to a productivity shock. This feature will be important when we study optimal policy under incomplete information in the next section.

3.4.2. Reduced form feedback rules

In this section we wish to investigate whether monetary policy derived from optimizing behavior gives rise to estimated behavior that resembles a Taylor rule. Surprisingly, estimation

[10] Our result are therefore little different from what occurs in a model where money enters separably. This is consistent with McCallum (2000), who shows that for reasonably calibrated money demand behavior, separability is not a bad approximation.

of policy rules with model-generated data yields statistical relationships that are somewhat similar to policy rules estimated for the U.S. economy, in the sense that monetary policy apparently increases interest rates in response to inflation, and that policy apparently smooths the behavior of interest rates. We present two examples, first the policy rule (T) based on Taylor (1993) and second the rule (CGG) based on Clarida, Gali, and Gertler (2000):

$$\begin{aligned}
\log R_t^{(T)} &= \underset{(0.01)}{0.13} + \underset{(0.12)}{0.20} \log\left(\frac{P_t}{P_{t-4}}\right) - \underset{(0.03)}{0.47} \log y_t + \underset{(0.04)}{0.16} \log R_{t-1} - \underset{(0.001)}{0.001} \log\left(\frac{M_t}{M_{t-1}}\right) \\
\log R_t^{(CGG)} &= \underset{(0.01)}{0.09} + \underset{(0.60)}{1.74} E_{t-1} \log\left(\frac{P_{t+1}}{P_t}\right) - \underset{(0.04)}{0.39} E_{t-1} \log y_t + \underset{(0.10)}{0.19} \log R_{t-1} - \underset{(0.004)}{0.014} \log\left(\frac{M_t}{M_{t-1}}\right).
\end{aligned}$$

The regression coefficients represent averages of 200 regressions run on 100 quarters worth of data, with standard deviations in parentheses. Our model does not contain a trend, we therefore do not correct output by a potential output measure to obtain an output gap measure as is usual in the literature on policy equations[11]. For specification (CGG) we estimate a bivariate VAR with output and prices for each sample, and use the VAR to generate one-quarter-ahead forecasts of inflation and output, with standard deviations in parentheses.

The qualitative properties of the two specifications are very similar. In both specifications output appears with the "wrong" sign, the coefficient on inflation has the "correct" sign, and the coefficient on output tends to be more significant than the coefficient on the inflation rate. The positive coefficient on inflation reflects the fact that under optmal policy the interest rate and inflation behave similarly to technology and money demand disturbances. The negative coefficient on output reflects the fact that optimal policy lowers the nominal interest rate relatively aggressively when there is a positive shock to technology. Hence, output increases are inversely related to the interest rate. Both specifications appear to indicate a desire for interest rate smoothing, which reflects the high degree of autocorrelation in the nominal interest rate. This autocorrelation is a result of the autocorrelation in the shock processes and not of any fundamental desire of the monetary authority to smooth rates[12]. For the (CGG) specification, the regressions seems to indicate that the behavior of nominal money is important for monetary policy: nominal money growth enters with a significant negative coefficient.

[11] For each specification quarterly output, the nominal interest rate, and growth rates are annualized. For specification (T) inflation is the average inflation over the last four quarters.

[12] Model-generated data indicate that output, inflation, and the nominal interest rate are all highly autocorrelated. Their first-order autocorrelation coefficients are 0.87, 0.95, and 0.90, respectively. The high autocorrelation is due to the highly autocorrelated shocks.

4. Optimal monetary policy with incomplete information

We have seen that optimal monetary policy is unresponsive to the behavior of money when the monetary authority has full information about the state of the economy. It is unlikely, however, that the monetary authority ever sees technology or money demand shocks, nor does it observe the true price distribution. In an environment where the policymaker does not have complete information on the state of the economy, optimal policy may respond to money because the behavior of money contains information about the unobserved state.

We model the notion of money as a signal for an information-constrained policymaker who never observes the true state of the economy as follows. In each period the policymaker receives two signals: a signal s_{0t} on money at the beginning of the period, contemporaneously with the decision to be made, and a signal s_{1t} on output at the end of the period, after a decision has been made. Given our linear approximation of the economy, the policymaker's information about the state x_t is summarized by his conditional expectations. The policymaker starts out with an expectation about the current state $\bar{x}_{t|t-1}$ conditional on information from past periods. Given the signal s_{0t} the policymaker updates the expectation to $\bar{x}_{t|t}$. After the equilibrium has been determined the policymaker receives the signal s_{1t} and he updates his conditional expectation to $\bar{x}_{t|t+}$. Using the law of motion for state variables he also forms expectations about next period's state to $\bar{x}_{t+1|t}$. We continue to assume that private agents have full information. We do so largely to make the problem more tractable.[13]

We show that certainty equivalence holds in the sense that the policymaker's decision depends only on his expected value of the state variables, $R_t = F_t \bar{x}_{t|t}$.[14] We can therefore use the results on optimal full-information policy rules from the previous section, and simply replace actual values of state variables with the policymaker's conditional expectations. The equilibrium values of all nonpredetermined variables will depend on the actual state and the policymaker's conditional expectation of the state $y_t = G_{1,t} x_t + G_{2,t} \bar{x}_{t|t}$. We solve the inference problem for the policymaker using a methodology based on the Kalman-filter. The inference problem is fairly complicated and the equilibrium outcome depends on the policymaker's policy rule. Thus, the separation of inference from optimization which Svensson and Woodford (2000) have shown to hold under common incomplete information is no longer

[13] As pointed out in Aoki (2000), private agents may face measurement problems less serious than those that face the central bank. Firms may have a better idea of the state of technology and consumers may not need to know the price of all goods when optimizing. Thus modeling private agents as having information superior to that possessed by the monetary authority seems a reasonable strategy. Even so, our information assumptions are severe.

[14] A detailed explanation of our procedure is contained in the appendix. Similar derivations can be found in Svensson and Woodford (2001).

present.

In most developed countries data on real GDP, nominal GDP, and the money stock are readily available for a policymaker. Furthermore, reliable information on money is available earlier than is information on GDP. We represent this information structure in our model as follows.[15] At the beginning of the period, before the policy instrument is set, the policymaker receives a signal on real balances $\ln(M_t/P_t)^o + \eta_R R_t = y_t + z_{mt} + \upsilon_{mt}$, where υ_{mt} is the measurement error. In reality policymakers observe separate signals on the nominal money stock and the price level. To keep the problem tractable, we have combined both signals into the real balance signal, which conveys more information than would the nominal money stock alone. At the end of period, after the policy instrument has already been determined, the policymaker receives a signal on real output, $y_t^o = y_t + \upsilon_{yt}$, where υ_{yt} is the measurement error on real output. The standard deviation of the measurement error of the nominal disturbance is 0.003, and the one for the output signal is 0.005. These values are consistent with the variance of revisions for real GDP and nominal M1.[16] Using these two signals, the monetary authority forms expectations of the economy's current state and conducts policy accordingly.

4.1. The behavior of the economy under incomplete information

We first study how the information structure affects the response of monetary policy to productivity shocks. We find that when the policymaker has only lagged information on output, a productivity shock results in an initial decline of output with subsequent hump-shaped convergence towards the full-information path. If the policymakers receives additional contemporaneous information on money, the policymaker can respond to the productivity shock and the initial negative effect on output is weakened. The more stable money demand is, the better can the policymaker respond to a productivity shock. We then show that with a stable money demand the policymaker is more likely to confuse money demand disturbances for productivity shocks. Since the policymaker responds strongly to perceived productivity shocks, this introduces undesirable volatility into the economy. For our example, having more information can reduce welfare. Finally, we estimate Taylor rules for our model economy and find that money growth enters significantly with a negative coefficient in these equations. For

[15] We assume that these measures are never updated, and that the authority never observes the actual state variables. For an alternative information setting in which the monetary authority observes the true state with a one period lag, see Aoki (2000) and Swensson and Woodford (2001).

[16] Because we parameterize the "nominal" noise using only the standard deviation of the money stock noise, we underestimate the magnitude of the "nominal" noise, implicitely giving money more signal value than it actually has.

the analysis of the model economy under incomplete information, we maintain the calibration of the previous section.

4.1.1. The effects of a productivity shock

Figure 4 displays the economy's response to a productivity shock: the actual values and the policymaker's expectations of state variables in panel A, and the actual values of non-predetermined variables in panel B. Because the policymaker has no contemporaneous information, there is no immediate response to the shock. From the standpoint of the full information case this inaction means that the nominal rate is roughly 50 basis points too high. With a one-period delay, the policymaker becomes aware of the productivity increase and understands that his inaction has resulted in a decline in q_1. In response to this new information, the policymaker now drastically lowers the nominal interest rate, but brings it back up to the full information case within three periods. We can see how incomplete information magnifies the policymaker's response to a shock: relative to full information the nominal interest rate movements are about four times as big. In turn, the policymaker's delayed and magnified response also introduces additional output and employment volatility. Firms who adjust their price in the period that the shock occurs know that it will take time for output to rise. Hence the path of wages and marginal cost will be lower than under full information and as a result firms slash prices. This price reduction reduces the anticipated inflation rate, and since the nominal interest rate is unchanged, it increases the real rate of interest. Although, future consumption is expected to increase, after the policymaker rectifies the mistake, the real rate increase is sufficient to induce a reduction in current-period consumption and output.

Could the excess volatility that occurs under incomplete information relative to what occurs under full information be reduced if the policymaker were to obtain contemporaneous information on money in addition to lagged information on output? It turns out that for our parameterization of money demand uncertainty, the nominal money stock does not contain much useful information about output. The impulse response functions with contemporaneous information are essentially the same as the ones in Figure 4. Basically, money demand is too volatile for observations on money to be of much value. Since observed movements in money can be largely attributed to the money demand shock, the inference of the technology shock and of relative prices is unchanged, and observing money does not improve the policymaker's knowledge of the state of the economy.

The property that money does not contain any useful information about output and hence technology disturbances is an empirical matter. If money demand is more stable, for example if we reduce the variance of the money demand shock by a factor of ten, the outcome is quite different (see Figure 5). In this situation contemporaneous money is sending a much sharper signal of contemporaneous output, and following a productivity shock the policymaker attributes much of the movement in money to that shock. The policymaker, however, also attributes some of the increased output to relatively low normalized lagged prices and a greater degree of inherited efficiency. The combination of beliefs leads to a decline in the nominal interest rate of 50 basis points, almost the identical decline as under full information. Yet output does not respond in nearly the same way as it does under full information. The key to the difference is subtle and involves the fact that the policymaker only gradually learns the true nature of the shock, but that the private sector already anticipates further nominal rate reductions induced by this learning process. As the policymaker learns more about the productivity shock, he implements further substantial nominal rate reductions that increase output further and reduce prices. The anticipation of big price reductions in the future induces current producers to reduce their prices substantially, and the expected inflation rate declines and the real rate increases. The real rate increase is then consistent with the hump-shaped response of output.

4.1.2. The effects of a money demand shock

Money demand shocks do not play an important role under full information. We would then expect that in an incomplete information environment a policymaker might ignore money demand shocks alltogether. This is actually the case when the policymaker has only lagged information on output.[17] We have also seen that the policymaker's response to a productivity shock is improved when he has contemporaneous information on money and money demand volatility is low. However, more information is not necessarily welfare improving in our environment. When the policymaker believes that the contemporaneous signal is a more reliable indicator of productivity shocks, the consequences are more severe when he mistakes a money demand shock for a productivity shock.

Figure 6 shows the effects of a money demand shock when the central bank observes money and money demand is stable; that is the variance of the money demand shock is one-tenth of our baseline specification. With this specification, the policymaker believes not only that a money demand shock has occurred, but also believes that the economy has been subject to a negative technology disturbance. This latter result occurs, because the inference problem and the policy response are no longer separate. Note that the contemporaneous

[17] We do not report the results here, but in this case the policy maker never updates his estimate of the money demand shock, and he responds only to perceived productivity and normalized lagged price shocks.

signal contains the sum of the money demand shock and the output response ($c_t + z_{mt} + \upsilon_{mt} < 0$). In our case the planner sees a negative signal, which he interprets as a negative productivity shock and a positive money demand shock. The policymaker ignores the money demand shock, but responds to the productivity shock and increases the nominal rate; the higher interest rate, in turn, reduces consumption, which leads to a negative signal. Thus the response and the inference are internally consistent. Unlike the case when the shock is actually a productivity shock, more information is not preferable and less variability in money demand actually inhibits accurate inference of the true disturbance.

Simulations of the model economies show that the economy with contemporaneous observation on money and low money demand volatility is much more volatile than the economy with information on lagged output only. This indicates that more information is not necessarily welfare improving in our environment.[18] We can verify this for our linear-quadratic approximations of the utility functions. In particular, we calculate the consumption equivalent welfare loss relative to full information (1) with lagged output only as 0.0072 percent of steady state consumption; (2) with contemporaneous money and high money demand volatility as 0.0058 percent; and (3) with contemporaneous money and low money demand volatility as 0.0713 percent.

4.1.3. Reduced form feedback rules

Policy rules estimated from data generated by a model where the policymaker has incomplete information show a policymaker who is more responsive to money stock changes. When we estimate the two equations (T) and (CGG) from section 3.4.2 for the baseline parameterization of the incomplete information model with a contemporaneous money signal and a lagged output signal, we obtain

$$\begin{aligned}
\log R_t^{(T)} &= \underset{(0.02)}{0.14} + \underset{(0.086)}{0.10} \log\left(\frac{P_t}{P_{t-4}}\right) - \underset{(0.17)}{0.76} \log y_t + \underset{(0.10)}{0.33} \log R_{t-1} - \underset{(0.02)}{0.11} \log\left(\frac{M_t}{M_{t-1}}\right) \\
\log R_t^{(CGG)} &= \underset{(0.02)}{0.12} + \underset{(0.20)}{1.75} E_{t-1} \log\left(\frac{P_{t+1}}{P_t}\right) - \underset{(0.16)}{0.65} E_{t-1} \log y_t - \underset{(0.09)}{0.15} \log R_{t-1} - \underset{(0.02)}{0.06} \log\left(\frac{M_t}{M_{t-1}}\right).
\end{aligned}$$

The Taylor specification (T) now displays a weaker response to annual inflation, a stronger negative response to the output gap, more interest rate smoothing, and a significant negative response to money growth. The Clarida-Gali-Gertler specification displays an unchanged positive response to predicted inflation, a stronger negative response to the predicted output

[18] This is not an entirely new observation. Pearlman (1992) has pointed out that more information need not be welfare improving in optimal control problems with partial information, even when the inference and the control problem are separable.

gap, no significant interest rate effect, and also a statistically negative dependence on money growth.

Incomplete information makes the dynamics of the model sufficienctly complicated such that the simple policy rules no longer capture the dynamics of the interest rate so well.[19] The increased importance of the model's internal propagation relative to the exogenous productivity shock is also reflected in the reduced persistence of variables. The first-order serial correlation coefficients on output, inflation, and the nominal interest rate are now .63, .66, and .51. Thus incomplete information and its subsequent effect on policy has a significant effect on how an econometrician would view this economy. This is true even though the private sector's behavior is unchanged.

5. Conclusion

In this paper we have attempted to evaluate how useful money is for the pursuit of monetary policy. We have done so for an environment where we have explicitly specified why money has real effects (sticky prices due to staggered price setting), and what are the policymaker's objective (maximize the expected utility of the representative agent) and constraints (choose a time-consistent policy rule). We have found that even though money communicates information on aggregate output, it is of limited use for a policymaker. We should emphasize that money's usefulness as a signal is an empirical matter, and that if the money demand was more stable than it appears to be, the value of money as a signal could dramatically increase. In particular money would be a useful signal in an environment driven by productivity shocks, but using it as a signal would have adverse consequences in the presence of money-demand disturbances. This finding suggests that time variation in the behavior of money-demand disturbances and in the types of shocks primarily affecting the economy could imply time variation in a policymaker's responsiveness to money.

Importantly, however, the dynamic behavior of an economy can be very different depending on what information is available to the policymaker. The dynamic behavior differs because policy responds very differently under full information from the way it responds under incomplete information. As stressed in Dotsey (1999), the form of the policy rule can be crucial in the type of model studied here. Thus, it may pay to model the economy's information structure accurately if one is to explain economic behavior. Information and signal extraction problems were a centerpiece of the early literature on rational expectations, these same items may again prove to be important for the analysis of optimal monetary policy.

[19] The average adjusted R^2 for the two specifications are 0.73 (0.05) for (T) and 0.85 (0.04) for (CGG). Under complete information they were 1.03(0.01) and .91(.05), respectively.

Finally, the joint specification of private pricing decisions and discretionary policy tends to produce somewhat high steady state rates of inflation. It would be interesting to relax both of these features of the model by allowing for state-dependent pricing decisions or some form of partial commitment. We hope to investigate these avenues in future work.

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Technical Appendix

A1. Introduction

This appendix describes the algorithms we use to find the steady state of an economy with discretionary optimal policy and incomplete information. We first describe a Markov-perfect equilibrium for the optimal control problem when the policymaker cannot commit to future policy choices. We describe a simple algorithm to find a linear approximation to the equilibrium. For this case we assume that the policymaker has complete information. We then consider the case where the policymaker has incomplete information, in particular he has less information than the private sector. This analysis takes as a starting point the linear approximation of the environment. The description of the linear-quadratic optimal control and estimation under incomplete information essentially follows Svensson and Woodford (2000). The section on incomplete information extends SW to the case of asymmetric information between the private sector and the policymaker.

A2. The model with full information

In this section we describe the optimal control problem of a policymaker under full information. The policymaker chooses allocations which satisfy the constraints that support the outcome of a market equilibrium given the actions of the policymaker. The policymaker maximizes an intertemporal objective function. For our applications this objective function is the expected present value of the representative agent's utility. We assume that the policymaker cannot commit to future policy choices, therefore we study a Markov-perfect equilibrium.

The constraints of the policymaker, are the first order conditions and market clearing conditions of the competitive equilibrium. It is useful to divide these constraints into two blocks: one that contains the evolution of the predetermined state variables, x, and denoted C_x, and the other that involves the non-predetermined flow variables, y, and denoted C_y. Formally, the constraints can be represented by

$$x_{t+1} = C_x\left(x_t, y_t, R_t, u_{x,t+1}\right) \tag{5.1}$$

$$E_t C_{y1}\left(x_{t+1}, y_{t+1}, R_{t+1}\right) = C_{y0}\left(x_t, y_t, R_t\right) \tag{5.2}$$

where R denotes the policy instruments and u is an iid random variable with mean zero. There are n_x state variables, n_y flow variables, and n_R instruments. Define $Z_t \equiv [x_t', y_t', R_t']'$. Equation (5.1) defines the law of motion for the state variables, and equation (5.2) reflects the fact that the chosen allocation has to satisfy the private sector's optimality conditions in a market economy. The function C_x is vector-valued of dimension n_x, C_{y0} and C_{y1} are vector-valued of dimension n_y, and E_t denotes the private sector's expectations conditional on the information set I_t. In this section we assume that the policymaker and the private sector have complete information at time t, $I_t = \{Z_\tau : \tau \leq t\}$. Note that (5.1) and (5.2) define an incomplete dynamic system, since the policymaker's decision rule with respect to the policy variables has not been specified. If we were to specify the policy variables as a function of the state and/or flow variables, such as in a Taylor-rule, we could solve (5.1) and (5.2) for the implied rational expectations equilibrium.

The objective function of the policymaker is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(Z_t) \tag{5.3}$$

where $0 < \beta < 1$. In our example U is the utility function of the representative agent, which generates the competitive equilibrium. We are looking for a time-consistent policy. For this purpose we solve for the set of Markov-perfect equilibria where the policymaker's decision depends only on pay-off relevant state variables, that is

$$R_t = F_t(x_t) \tag{5.4}$$

and the equilibrium outcome is such that the flow variables depend only on the current state

$$y_t = G_t(x_t). \tag{5.5}$$

For the Markov-perfect equilibrium, the policymaker takes next period's outcome functions F_{t+1} and G_{t+1} and the continuation value function $V_{t+1}(x_{t+1})$ as given, and the optimization problem is

$$\begin{aligned} V_t(x_t) &= \max_{x_{t+1}, y_t, R_t} E_t\left[U(x_t, y_t, R_t) + \beta V_{t+1}(x_{t+1})\right] && (5.6)\\ & \quad \text{s.t. } x_{t+1} = C_x(x_t, y_t, R_t, u_{x,t+1}) && (5.7)\\ & \quad E_t\left[C_{y1}(x_{t+1}, G_{t+1}(x_{t+1}), F_{t+1}(x_{t+1}))\right] = C_{y0}(x_t, y_t, R_t). && (5.8) \end{aligned}$$

A Markov-perfect equilibrium is characterized by the triple (F, G, V) such that (5.6), (5.7), and (5.8) maps $(F_{t+1}, G_{t+1}, V_{t+1}) = (F, G, V)$ to $(F_t, G_t, V_t) = (F, G, V)$.

For the solution we proceed in several steps. We first construct a linear-quadratic approximation of the problem around a steady state indexed by the policy instrument R^*. We then solve the LQ approximation, and derive the steady state of the approximation R^*_{LQ}. This defines a mapping from R^* to R^*_{LQ}. We solve for a steady state choice of policy instruments such that $R^* = R^*_{LQ}$.

Solving for an approximate steady state

Step 1. Suppose the steady state values of the policy instruments are given by R^*. Conditional on R^* we can solve (5.1) and (5.2) for the steady state values of the state and flow variables (x^*, y^*). Now derive a linear approximation of the constraints (5.1) and (5.2)

$$x_{t+1} = C_{x,Z} Z_t + u_{x,t+1} \tag{5.9}$$

$$E_t \left[C_{y1,Z} Z_{t+1} \right] = C_{y0,Z} Z_t \tag{5.10}$$

and a quadratic approximation of the period utility function[20]

$$Z_t' \mathbf{U} Z_t = [x_t', y_t', R_t'] \begin{bmatrix} U_{xx} & U_{xy} & U_{xz} \\ U_{yx} & U_{yy} & U_{yz} \\ U_{zx} & U_{zy} & U_{zz} \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ R_t \end{bmatrix}.$$

Step 2. Given the linear-quadratic structure, we guess that next period's non-predetermined variables are linear functions of next period's state variable and that the continuation value from next period on is a quadratic function of next period's state variable. Specifically,

$$R_{t+1} = F_{t+1} x_{t+1} \tag{5.11}$$

$$y_{t+1} = G_{t+1} x_{t+1} \tag{5.12}$$

and $x_{t+1}' V_{t+1} x_{t+1}$ is the continuation value.

We now determine the equilibrium outcome for the current period and next period, conditional on the current state and policy choice. Assume that the matrix

$$C_{1,t} = \begin{bmatrix} I_x & 0_{xy} \\ C_{y1,x} + C_{y1,R} F_{t+1} & C_{y1,y} \end{bmatrix}$$

is invertible, then we can rewrite the constraints as

$$\begin{bmatrix} x_{t+1} \\ E_t y_{t+1} \end{bmatrix} = C_{1,t}^{-1} \begin{bmatrix} C_x \\ C_{y0,Z} \end{bmatrix} Z_t = \begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx,t} & A_{yy,t} \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} + \begin{bmatrix} B_x \\ B_{y,t} \end{bmatrix} R_t + \begin{bmatrix} u_{x,t+1} \\ 0 \end{bmatrix}. \tag{5.13}$$

We now proceed as in Svensson and Woodford (2000). Substitute the transition equations for the state variables (5.13) in our guess for next period's flow variables (5.12) and take conditional expectations

$$E_t \left[y_{t+1} \right] = G_{t+1} \left(A_{xx} x_t + A_{xy} y_t + B_x R_t \right).$$

[20] In the following we will interpret the variables in terms of deviations from the steady state. Furthermore, the first derivative of the utility function is implicitly included by adding a constant term to the vector of state variables $[1, x']'$. We also have normalized the covariance matrix of u_x such that the derivative of $C_{x,u_x} = I$.

Now substitute these expectation for next period's flow variables in the optimality conditions from the competitive equilibrium (5.13) and get

$$G_{t+1}\left(A_{xx}x_t + A_{xy}y_t + B_x R_t\right) = A_{yx,t}x_t + A_{yy,t}y_t + B_{y,t}R_t.$$

Assuming that $A_{yy,t} - G_{t+1}A_{xy}$ is invertible, we can solve for this period's flow variables

$$\begin{aligned} y_t &= \tilde{A}_t x_t + \tilde{B}_t R_t \text{ with} \\ \tilde{A}_t &\equiv \left(A_{yy,t} - G_{t+1}A_{xy}\right)^{-1}\left(G_{t+1}A_{xx} - A_{yx,t}\right) \\ \tilde{B}_t &\equiv \left(A_{yy,t} - G_{t+1}A_{xy}\right)^{-1}\left(G_{t+1}B_x - B_{y,t}\right). \end{aligned} \tag{5.14}$$

Substituting (5.14) for this period's flow variables in the transition equation (5.9) yields

$$\begin{aligned} x_{t+1} &= A_t^* x_t + B_t^* R_t + u_{x,t+1} \text{ with} \\ A_t^* &\equiv A_{xx} + A_{xy}\tilde{A}_t \\ B_t^* &\equiv B_x + A_{xy,t}B_t. \end{aligned} \tag{5.15}$$

After substituting for y_t from (5.14) and for x_{t+1} from (5.15) in the period t optimization problem

$$\max\left\{Z_t'\mathbf{U}Z_t + \beta E_t\left[x_{t+1}'V_{t+1}x_{t+1}\right]\right\} \text{ s.t. (5.14) and (5.15).}$$

we get

$$\begin{aligned} &\max_{R_t}\left[x_t', R_t'\right]\begin{bmatrix} Q_{xx,t} & Q_{xz,t} \\ Q_{zx,t} & Q_{zz,t} \end{bmatrix}\begin{bmatrix} x_t \\ R_t \end{bmatrix} \\ &+\beta E\left[\left(A_t^*x_t + B_t^*R_t + u_{x,t+1}\right)' V_{t+1}\left(A_t^*x_t + B_t^*R_t + u_{x,t+1}\right)\right] \end{aligned}$$

with

$$Q_{xx,t} = \left[I_x, \tilde{A}_t'\right]\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} I_x \\ \tilde{A}_t \end{bmatrix} \tag{5.16}$$

$$Q_{xz,t} = \left[I_x, \tilde{A}_t'\right]\left\{\begin{bmatrix} U_{xz} \\ U_{yz} \end{bmatrix} + \begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} 0 \\ \tilde{B}_t \end{bmatrix}\right\}$$

$$\begin{aligned} Q_{zz,t} &= U_{zz} + \left[0, \tilde{B}_t\right]\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix}\begin{bmatrix} 0 \\ \tilde{B}_t \end{bmatrix} + \left[0, \tilde{B}_t'\right]\begin{bmatrix} U_{xz} \\ U_{yz} \end{bmatrix} \\ &\quad + \left[U_{zx}, U_{zy}\right]\begin{bmatrix} 0 \\ \tilde{B}_t \end{bmatrix}. \end{aligned} \tag{5.17}$$

The first order condition for the optimal choice of the policy instrument is

$$0 = \left(Q_{zz,t} + \beta B_t^{*\prime}V_{t+1}B_t^*\right)R_t + \left(Q_{zx,t} + \beta B_t^{*\prime}V_{t+1}A_t^{*\prime}\right)x_t$$

Assuming that $Q_{zz,t} + \beta B_t^{*\prime} V_{t+1} B_t^*$ is invertible we can solve for the policy instrument and define this period's policy function $R_t = F_t x_t$ and flow variable equation $y_t = G_t x_t$ with

$$F_t = -\left(Q_{zz,t} + \beta B_t^{*\prime} V_{t+1} B_t^*\right)^{-1} \left(Q_{zx,t} + \beta B_t^{*\prime} V_{t+1} A_t^{*\prime}\right) \tag{5.18}$$
$$G_t = \tilde{A}_t + \tilde{B}_t F_t. \tag{5.19}$$

Substituting for the policy function in (5.16) yields the current period value as a quadratic function of the current period state defined by the matrix

$$V_t = [I, F_t'] \left\{ Q_t + \beta \begin{bmatrix} A_t^{*\prime} \\ B_t^{*\prime} \end{bmatrix} V_{t+1} [A_t^*, B_t^*] \right\} \begin{bmatrix} I \\ F_t \end{bmatrix} \tag{5.20}$$

A stationary Markov-perfect equilibrium to the policymaker's decision problem is a triple of matrices (F, G, V) which is a fixed point of the mapping defined by equations (5.13) through (5.20).

Step 3. Calculate the steady state of the Markov-perfect equilibrium. Substituting the policy rule F and the equilibrium function G into the transition equation for the state variables we get

$$x_{LQ}^* = A_{xx} x_{LQ}^* + A_{xy} G x_{LQ}^* + B_x F x_{LQ}^*.$$

Recall that constant terms are implicit in this equation through the definition of the state variable which contains a constant term. We can solve this expression for the steady state value of the linear approximation x_{LQ}^* and $R_{LQ}^* = F x_{LQ}^*$. Since we started with the assumption that the steady state of the Markov perfect equilibrium is R^* we adjust R^* until $R_{LQ}^* = R^*$.

Implementation of the algorithm

The implementation of the algorithm is straightforward, with two minor exceptions. The derivation of the Markov-perfect equilibrium suggests that we use a simple iteration scheme: given an assumption on the triple $(F_{t+1}, G_{t+1}, V_{t+1})$ use equations (5.13) through (5.20) to obtain values for the triple (F_t, G_t, V_t), and iterate until convergence. There are two problems with simple iterations, which relate to the fact that we have not shown that such a process will indeed converge.

First, we have found that frequently we obtain convergence on the linear terms of the functions F, G, and V, but we cannot obtain convergence once we include the constant terms in these functions. This problem is easily dealt with. Given the linear quadratic structure we know that the linear terms are independent of the constant terms, and in a first run we ignore the constant terms and interpret the model in terms of deviations from the steady state. Essentially this means ignoring the linear terms of the utility function. Usually we obtain convergence on the linear terms for this simplified model. After we have obtained the linear terms, the constant terms can be obtained as a solution to a linear system of equations. We need to know the constant terms since they will determine the steady state of the approximation economy.

The second problem is that occasionally the simple iteration scheme does not converge for the linear terms. In this case we solve the equations (5.13) through (5.20) as one big system of non-linear equations. This procedure tends to be slower than simple iterations and good starting values are required.

A3. The model with incomplete and asymmetric information

We now consider a special case of incomplete information: we assume that the private sector of the economy has complete information, whereas the policymaker has incomplete information. In particular, we assume that the policymaker receives two types of signals about the economy. The first signal s_{0t} is received at the beginning of the period, and its value is determined simultaneously with the values of the flow variables and the policy instrument. The second signal s_{1t} is received at the end of the period, after the flow variables and the policy instrument have been determined. We call the information contained in the first and second signal contemporaneous respectively lagged. The signals are related to the true state of the economy according to

$$s_{it} = S_i\left[x_t', y_t'\right]' + e_{it} \tag{5.21}$$

where e_{it} is iid with mean zero and covariance matrix Σ_{ei}, for $i = 0, 1$. The policymaker's information set at the beginning of period t is $\bar{I}_t = \{(R_t, s_{0t})$ and $(R_\tau, s_{0\tau}, s_{1\tau})$ for $\tau < t\}$ and at the end of the period it is $\bar{I}_t^+ = \bar{I}_t \cup \{s_{1t}\}$. We denote the policymaker's expectations about the current state of the economy conditional on the available information as $\bar{x}_{t|t} = E\left[x_t|\bar{I}_t\right]$, respectively $\bar{x}_{t+1|t} = E\left[x_{t+1}|\bar{I}_t^+\right]$. The private sector has complete information, that is the beginning and end-of-period information sets are $I_t = \{(x_t, y_t, z_t, s_{0t})$ and $(x_\tau, y_\tau, z_\tau, s_{0\tau}, s_{1\tau})$ for $\tau < t\}$ and $I_t^+ = I_t \cup \{s_{1t}\}$, respectively, and the conditional expectations are denoted $x_{t|t}$ and $x_{t+1|t}$.

The optimal control problem.

We guess that next period's policymaker's decision is

$$R_{t+1} = F_{t+1}\bar{x}_{t+1|t+1} \tag{5.22}$$

and that next period's flow variables depend on the actual values of the state variables and the policymaker's perceptions of the state variables

$$y_{t+1} = G_{1,t+1}x_{t+1} + G_{2,t+1}\bar{x}_{t+1|t+1}. \tag{5.23}$$

Also assume that the continuation value of next period's policymaker is given by

$$E\left[x'_{t+1}V_{t+1}x_{t+1}|\bar{I}_{t+1}\right].$$

When the policymaker forecasts next period's flow variables, the distinction between actual and forecasted variables is irrelevant

$$\bar{y}_{t+1|t} = G_{t+1}\bar{x}_{t+1|t} \text{ and } G_{t+1} \equiv G_{1,t+1} + G_{2,t+1}.$$

Proceeding as before we can obtain the policymaker's expectation about the current flow variables

$$\bar{y}_{t|t} = \tilde{A}_t\bar{x}_{t|t} + \tilde{B}_tR_t$$

and his forecast of next period's state variables

$$\bar{x}_{t+1|t} = A^*_t\bar{x}_{t|t} + B^*_tR_t$$

where $\tilde{A}_t$, $\tilde{B}_t$, A^*_t, and B^*_t are defined as in (5.15) and (5.14).

The policymaker's value function is now given by

$$E\left[Z'_t\mathbf{U}Z_t + \beta x'_{t+1}V_{t+1}x_{t+1} \mid \bar{I}_t\right] = \bar{Z}'_{t|t}\mathbf{U}\bar{Z}_{t|t} + \beta\bar{x}'_{t+1|t}V_{t+1}\bar{x}_{t+1|t} + l_t \tag{5.24}$$

where

$$l_t \equiv E\left[\left(Z_t - \bar{Z}_{t|t}\right)'\mathbf{U}\left(Z_t - \bar{Z}_{t|t}\right) + \beta\left(x_{t+1} - \bar{x}_{t+1|t}\right)'V_{t+1}\left(x_{t+1} - \bar{x}_{t+1|t}\right) \mid \bar{I}_t\right]. \tag{5.25}$$

Notice that if the policymaker's inference and forecast errors are independent of current policy, then the term l_t is independent of current policy. For this "certainty equivalence" case, optimization proceeds with respect to the policymaker's forecasts of the state variables, and we can use the results from the full information case where we replace actual values with the policymaker's best forecasts.

Inference: the Kalman filter

In this section we solve the optimal signal extraction problem while assuming that the policy is as in the full information case. From the solution to the optimal control problem we have $R_t = F_t\bar{x}_{t|t}$ and $\bar{y}_{t|t} = G_t\bar{x}_{t|t}$. In order to determine the realized values of the flow variables we also need the components of G_t as defined in (5.23). Below we will construct G_{1t} and G_{2t}, but for now we take them as given. In the following we construct the Kalman-filter recursively. The equilibrium we study in the paper is a fixed point of the mapping implied by this procedure.

We first use the policy rule (5.22) and the equilibrium flow variables equation (5.23) to eliminate R_t and y_t from the transition equation (5.13) and the signal equation (5.21):

$$\begin{aligned} x_{t+1} &= J_t \bar{x}_{t|t} + H_t x_t + u_{x,t+1} \text{ with} \\ H_t &\equiv A_{xx} + A_{xy} G_{1,t} \\ J_t &\equiv A_{xy} G_{2,t} + B_x F_t \end{aligned} \tag{5.26}$$

and

$$\begin{aligned} s_{it} &= M_{it} \bar{x}_{t|t} + L_{it} x_t + e_{i,t} \text{ with} \\ L_{it} &\equiv S_{ix} + S_{iy} G_{1,t} \\ M_{it} &\equiv S_{iy} G_{2,t} \end{aligned} \tag{5.27}$$

These equations essentially define the Kalman-filter problem: equation (5.26) is the transition equation for the unobserved state, and equation (5.27) is the measurement equation which relates the signals to the state. The only non-standard feature is that the policymaker's updated estimate of the state appears on the right-hand side of each equation. We now introduce the following notation for the derivation of the Kalman filter. First, define the forecast errors for the state and signals as

$$\begin{aligned} \tilde{x}_t &\equiv x_t - \bar{x}_{t|t-1} \text{ and } \tilde{s}_{0t} \equiv s_{0t} - \bar{s}_{0,t|t-1} \text{ at the beginning of the period, and} \\ \tilde{x}_t^+ &\equiv x_t - \bar{x}_{t|t} \quad \text{ and } \tilde{s}_{1t} \equiv s_{1t} - \bar{s}_{0,t|t} \quad \text{ at the end of the period.} \end{aligned}$$

The forecast update is then $\hat{x}_t = \bar{x}_{t|t} - \bar{x}_{t|t-1}$. Second, in the standard Kalman-filter the update of the state variable is a linear function of the signal forecast error. Here we follow Svensson and Woodford (2000) and guess that at the beginning of the period the policymaker's update of the state is a linear function of a transformation of the signal, $\tilde{s}_{0t} - M_{0t}\bar{x}_{t|t}$,

$$\hat{x}_t = K_{0t} \left(L_{0t} \tilde{x}_t + e_{0t} \right) \tag{5.28}$$

This guess will be verified below. We construct the Kalman-filter in two steps, for each signal separately.

Beginning of the period. From equation (5.27) and the law of iterated expectations, the policymaker's forecast of this period's contemporaneous signal, based on the information available at the end of last period, is

$$\bar{s}_{0,t|t-1} = \left(M_{0t} + L_{0t} \right) \bar{x}_{t|t-1}$$

We subtract this expression for the prior expectation of the contemporaneous signal from its actual value (5.27), and obtain the forecast error for the signal

$$\tilde{s}_{0t} = L_{0t}\left(x_t - \bar{x}_{t|t-1}\right) + M_{0t}\left(\bar{x}_{t|t} - \bar{x}_{t|t-1}\right) + e_{0,t} = L_{0t}\tilde{x}_t + M_{0t}\hat{x}_t + e_{0t}.$$

Substituting our guess (5.28) for $\hat{x}_t$, the expression for the contemporaneous forecast error simplifies to

$$\begin{aligned}
\tilde{s}_{0t} &= N_{0t}\tilde{x}_t + \nu_t && (5.29)\\
\text{with } N_{0t} &\equiv \left(I + M_{0t}K_{0t}\right)L_{0t}\\
\nu_t &\equiv \left(I + M_{0t}K_{0t}\right)e_{0,t} \text{ and}\\
\Sigma_{\nu,t} &\equiv E_t\left[\nu_t\nu_t'\right] = \left(I + M_{0t}K_{0t}\right)\Sigma_{e0}\left(I + M_{0t}K_{0t}\right)'.
\end{aligned}$$

From equation (5.29) we get the update of the state variable $\hat{x}_t$ as a linear projection of $\tilde{x}_t$ onto $\tilde{s}_{0t}$

$$\begin{aligned}
\hat{x}_t &= E\left[\tilde{x}_t\tilde{s}_{0t}'|\bar{I}_{t-1}^+\right]E\left[\tilde{s}_{0t}\tilde{s}_{0t}'|\bar{I}_{t-1}^+\right]^{-1}\tilde{s}_{0t} = \hat{K}_{0t}\tilde{s}_{0t} && (5.30)\\
\text{with } \hat{K}_{0t} &\equiv P_{t|t-1}N_{0t}'\left(N_{0t}P_{t|t-1}N_{0t}' + \Sigma_{\nu,t}\right)^{-1} \text{ and}\\
P_{t|t-1} &\equiv E\left[\tilde{x}_t\tilde{x}_t'|\bar{I}_{t-1}^+\right].
\end{aligned}$$

Based on the posterior of the state variable we get an expression for the state variable forecast error

$$\tilde{x}_t^+ = x_t - \bar{x}_{t|t} = x_t - \bar{x}_{t|t-1} - \hat{K}_{0t}\tilde{s}_t = \tilde{x}_t - \hat{K}_{0t}\left(N_{0t}\tilde{x}_t + \nu_t\right) \quad (5.31)$$

which we can use to get an estimate of the variance of the update error

$$P_{t|t} = E\left[\left(x_t - \bar{x}_{t|t}\right)\left(x_t - \bar{x}_{t|t}\right)'|\bar{I}_t\right] = \left(I - \hat{K}_{0t}N_{0t}\right)P_{t|t-1}. \quad (5.32)$$

We now have two expressions for how to use the contemporaneous signal to update the estimate of the state variables: our initial guess from equation (5.28) and the implied Kalman filter from equation (5.30). For our initial guess to be correct we need that

$$K_{0t} = \hat{K}_{0t}\left(I + M_{0t}K_{0t}\right). \quad (5.33)$$

Substituting for the definition of $\hat{K}_{0t}$ from (5.30) and for N_{0t} and $\Sigma_{v,t}$ from equation (5.29) and simplifying we get

$$K_{0t} = P_{t|t-1}L_{0t}'\left(L_{0t}P_{t|t-1}L_{0t}' + \Sigma_{e0}\right)^{-1}. \quad (5.34)$$

Finally note that (5.33) together with the definition of N_{0t} from (5.29) also implies that

$$K_{0t}L_{0t} = \hat{K}_{0t}N_{0t}. \quad (5.35)$$

End of the period. We now turn to the second stage when the policymaker receives signal s_{1t}. From equation (5.27) and the law of iterated expectations, the policymaker's forecast of this period's lagged signal based on the information available at the beginning of the period is

$$\bar{s}_{1,t|t} = (M_{1t} + L_{1t})\,\bar{x}_{t|t}.$$

We subtract this expression for the prior expectation of the lagged signal from its actual value (5.27), and obtain the forecast error for the signal

$$\tilde{s}_{1t} = L_{1t}\left(x_t - \bar{x}_{t|t}\right) + e_{1,t} = L_{1t}\tilde{x}_t^+ + e_{1t}. \tag{5.36}$$

From equation (5.36) we get the update of the state variable $\hat{x}_t^+$ as a linear projection of $\tilde{x}_t^+$ onto $\tilde{s}_{1t}$

$$\begin{aligned}
\hat{x}_t^+ &= E\left[\tilde{x}_t^+\tilde{s}_{1t}'|\bar{I}_t\right] E\left[\tilde{s}_{1t}\tilde{s}_{1t}'|\bar{I}_t\right]^{-1}\tilde{s}_{1t} = \hat{K}_{1t}\tilde{s}_{1t} \text{ with} \\
\hat{K}_{1t} &\equiv P_{t|t}L_{1t}'\left(L_{1t}P_{t|t}L_{1t}' + \Sigma_{e1}\right)^{-1} \text{ and} \\
P_{t|t} &\equiv E\left[\tilde{x}_t\tilde{x}_t'|\bar{I}_t\right].
\end{aligned} \tag{5.37}$$

The updated conditional expectation of this period's and next period's state variables then are

$$\begin{aligned}
\bar{x}_{t|t}^+ &= \bar{x}_{t|t}^+ + \hat{K}_{1t}\left\{s_{1t} - (M_{1t} + L_{1t})\,\bar{x}_{t|t}\right\} \\
\bar{x}_{t+1|t} &= J_t\bar{x}_{t|t} + H_t\bar{x}_{t|t}^+.
\end{aligned}$$

From this expression and the law of motion for state variables (5.26) we obtain the forecast error for the state variables at the beginning of next period as a function of this period's beginning-of-period forecast error

$$\begin{aligned}
\tilde{x}_{t+1} &= x_{t+1} - \bar{x}_{t+1|t} \\
&= H_t\left(x_t - \bar{x}_{t|t}^+\right) + u_{x,t+1} \\
&= H_t\left(I - \hat{K}_{1t}L_{1t}\right)\tilde{x}_t + u_{x,t+1} - H_t\hat{K}_{1t}e_{1t},
\end{aligned} \tag{5.38}$$

and we can use this expression to update the estimate of next period's beginning-of-period forecast error covariance matrix $P_{t+1|t} = E\left[\left(x_{t+1} - \bar{x}_{t+1|t}\right)\left(x_{t+1} - \bar{x}_{t+1|t}\right)'|\bar{I}_t^+\right]$

$$P_{t+1|t} = H_t\left(I - \hat{K}_{1t}L_{1t}\right)P_{t|t}\left(I - \hat{K}_{1t}L_{1t}\right)'H_t' + H_t\hat{K}_{1t}\Sigma_{e1}\hat{K}_{1t}'H_t' + \Sigma_{\mathrm{u}}. \tag{5.39}$$

It remains to show how the equilibrium function (5.23) can be defined.

The equilibrium relation G

Suppose the equilibrium rule is as defined in equation (5.23). Then the policymaker's and the private sector's forecast of next period's flow variables, and the implied difference in their forecasts is

$$\begin{aligned} \bar{y}_{t+1|t} &= \left(G_{1,t+1} + G_{2,t+1}\right) \bar{x}_{t+1|t} \\ y_{t+1|t} &= G_{1,t+1} x_{t+1|t} + G_{2,t+1} E\left[\bar{x}_{t+1|t+1} | I_t\right] \\ y_{t+1|t} - \bar{y}_{t+1|t} &= G_{1,t+1}\left(x_{t+1|t} - \bar{x}_{t+1|t}\right) + G_{2,t+1}\left(E\left[\bar{x}_{t+1|t+1}|I_t\right] - \bar{x}_{t+1|t}\right). \end{aligned} \tag{5.40}$$

We can write the policymaker's conditional expectation at the beginning of next period as this period's beginning-of-period conditional expectations plus the sum of this period's three updates

$$\bar{x}_{t+1|t+1} = \bar{x}_{t|t} + \left(\bar{x}^{+}_{t|t} - \bar{x}_{t|t}\right) + \left(\bar{x}_{t+1|t} - \bar{x}^{+}_{t|t}\right) + \left(\bar{x}_{t+1|t+1} - \bar{x}_{t+1|t}\right).$$

The public's expectations of these three updates are in turn

$$\begin{aligned} E\left[\bar{x}^{+}_{t|t} - \bar{x}_{t|t}|I_t\right] &= \hat{K}_{1,t} L_{1,t} \tilde{x}^{+}_t \\ E\left[\bar{x}_{t+1|t} - \bar{x}^{+}_{t|t}|I_t\right] &= \bar{x}_{t+1|t} - \bar{x}_{t|t} + \left(H_t - I\right) \hat{K}_{1,t} L_{1,t} \tilde{x}^{+}_t \\ E\left[\bar{x}_{t+1|t+1} - \bar{x}_{t+1|t}|I_t\right] &= \hat{K}_{0,t+1} N_{0,t+1} H_t \left(I - \hat{K}_{1,t} L_{1,t}\right) \tilde{x}^{+}_t. \end{aligned}$$

Substituting these expressions yields

$$E\left[\bar{x}_{t+1|t+1}|I_t\right] - \bar{x}_{t+1|t} = \Psi_t \tilde{x}^{+}_t \text{ with } \Psi_t = \left[H_t \hat{K}_{1,t} L_{1,t} + \hat{K}_{0,t+1} N_{0,t+1} H_t \left(I - \hat{K}_{1,t} L_{1,t}\right)\right]. \tag{5.41}$$

From the transition equation for the state variables (5.13), we get that the difference between the private sector's forecast and the policymaker's forecast of the state variables is

$$x_{t+1|t} - \bar{x}_{t+1|t} = A_{xx}\left(x_t - \bar{x}_{t|t}\right) + A_{xy}\left(y_t - \bar{y}_{t|t}\right). \tag{5.42}$$

Using (5.42) and (5.41) in (5.40) yields

$$y_{t+1|t} - \bar{y}_{t+1|t} = \left(G_{1,t+1} + G_{2,t+1}\Psi_t\right)\left(x_{t+1|t} - \bar{x}_{t+1|t}\right) + G_{1,t+1} A_{xy}\left(y_t - \bar{y}_{t|t}\right)$$

On the other hand, the optimality constraints on allocations (5.13), imply that the difference in forecasts is given by

$$y_{t+1|t} - \bar{y}_{t+1|t} = A_{yx,t}\left(x_t - \bar{x}_{t|t}\right) + A_{yy,t}\left(y_t - \bar{y}_{t|t}\right).$$

Equating the last two expressions and solving for the current value of the flow variables yields

$$y_t = \bar{y}_{t|t} + \left(A_{yy,t} - G_{1,t+1} A_{xy}\right)^{-1}\left(G_{1,t+1} A_{xx} + G_{2,t+1}\Psi_t - A_{yx,t}\right)\left(x_t - \bar{x}_{t|t}\right)$$

or

$$\begin{aligned} y_t &= G_{1,t}x_t + G_{2,t}\bar{x}_{t|t} \\ \text{where } G_{1,t} &\equiv \left(A_{yy,t} - G_{1,t+1}A_{xy}\right)^{-1}\left(G_{1,t+1}A_{xx} + G_{2,t+1}\Psi_t - A_{yx,t}\right) \\ G_{2,t} &\equiv G_t - G_{1,t}. \end{aligned} \tag{5.43}$$

Note that the matrix $G_{1,t}$ is also implicitly defined as

$$A_{yy,t}G_{1,t} + A_{yx,t} = G_{1,t+1}H_t + G_{2,t+1}\Psi_{t+1}. \tag{5.44}$$

The interaction of optimal control and inference

We have solved the optimal control problem under the assumption that certainty equivalence holds, that is the policymaker's decision is a function of the policymaker's expectation of the state of the economy, conditional on the available information at the time the decision is made. For this to be true it remains to be shown that the term l_t in the policymaker's value function (5.24) is indeed independent of the policy choice. From equation (5.25) it follows that l_t is a function of the update error $x_t - \bar{x}_{t|t}$,

$$\begin{aligned} x_t - \bar{x}_{t|t} &= \tilde{x}_t - \hat{x}_t \\ y_t - \bar{y}_{t|t} &= G_{1,t}\left(x_t - \bar{x}_{t|t}\right) \\ x_{t+1} - \bar{x}_{t+1|t} &= A_{xx}\left(x_t - \bar{x}_{t|t}\right) + A_{xy}\left(y_t - \bar{y}_{t|t}\right) + u_{t+1}, \end{aligned}$$

and it is enough to show that the update error for the state variable $x_t - \bar{x}_{t|t}$ is independent of policy. The update error has two components, the forecast error from the previous period $\tilde{x}_t$, and the update of the state variable estimate $\hat{x}_t$. As of time t the forecast error $\tilde{x}_t$ is independent of the current policy choice. This follows from equation (5.38).

Notice, that the only way the policy rule affects the signal extraction problem is through the determination of G_t in equation (5.19). Thus if we can show that the update of the state variable estimate does not depend on G_t, then the update error is independent of current policy. From equation (5.28) the time t update of the state variable depends on the matrices K_{0t} and L_{0t}, and the forecast error $\tilde{x}_t$. From equation (5.34), K_{0t} depends on $P_{t|t-1}$ and L_{0t}, and L_{0t} in turn depends on $G_{1,t}$. From equation (5.39), $P_{t|t-1}$ depends on past outcomes such as H_{t-1}, but not current choices, so it is independent of current policy. Thus we have to show that G_{1t} is independent of current policy. From the definition of G_{1t} in equations (5.41) and (5.43), together with the definition of H_t in (5.26), $\hat{K}_{1,t}L_{1t}$ in (5.37), L_{1t} in (5.27), and $P_{t|t}$ in (5.32) it follows that G_{1t} is a function of $G_{1,t+1}$ and G_{t+1}, but not of G_t. Therefore G_{1t} is independent of current policy F_t.

We have just shown that certainty equivalence holds, that is conditional on past and future decision rules, the inference problem of the current policymaker is not affected by the policymaker's own choices. This does not mean that the signal extraction problem is independent of the optimal control problem. Since the Markov-perfect equilibrium we study is a fixed point of the mapping defined by the inference problem, and since G is a function of the policy rule F, the inference problem cannot be separated from the control problem.

Table 1. Parameters and Steady State

			Parameters			
$\beta = 0.99$,	$\theta = 0.73$,	$\rho = -17.52$,	$\chi = 1.30$,	$\phi = 3.0$,	$\varepsilon = 10$,	$J = 4$,
	$\gamma_y = 0.9$,	$\gamma_m = 0.9$,	$\sigma_y = 0.01$,	$\sigma_m = 0.026$		
			Steady State			
$y = 0.265$,	$c = 0.265$,	$n = 0.265$,	$R = 1.036$,	$\hat{P} = 1.025$,	$w = 0.897$,	
	$q_0 = 1.042$,	$q_1 = 1.016$,	$q_2 = 0.991$,	$q_3 = 0.966$,		
	$y_0 = 0.175$,	$y_1 = 0.225$,	$y_2 = 0.289$,	$y_3 = 0.370$		

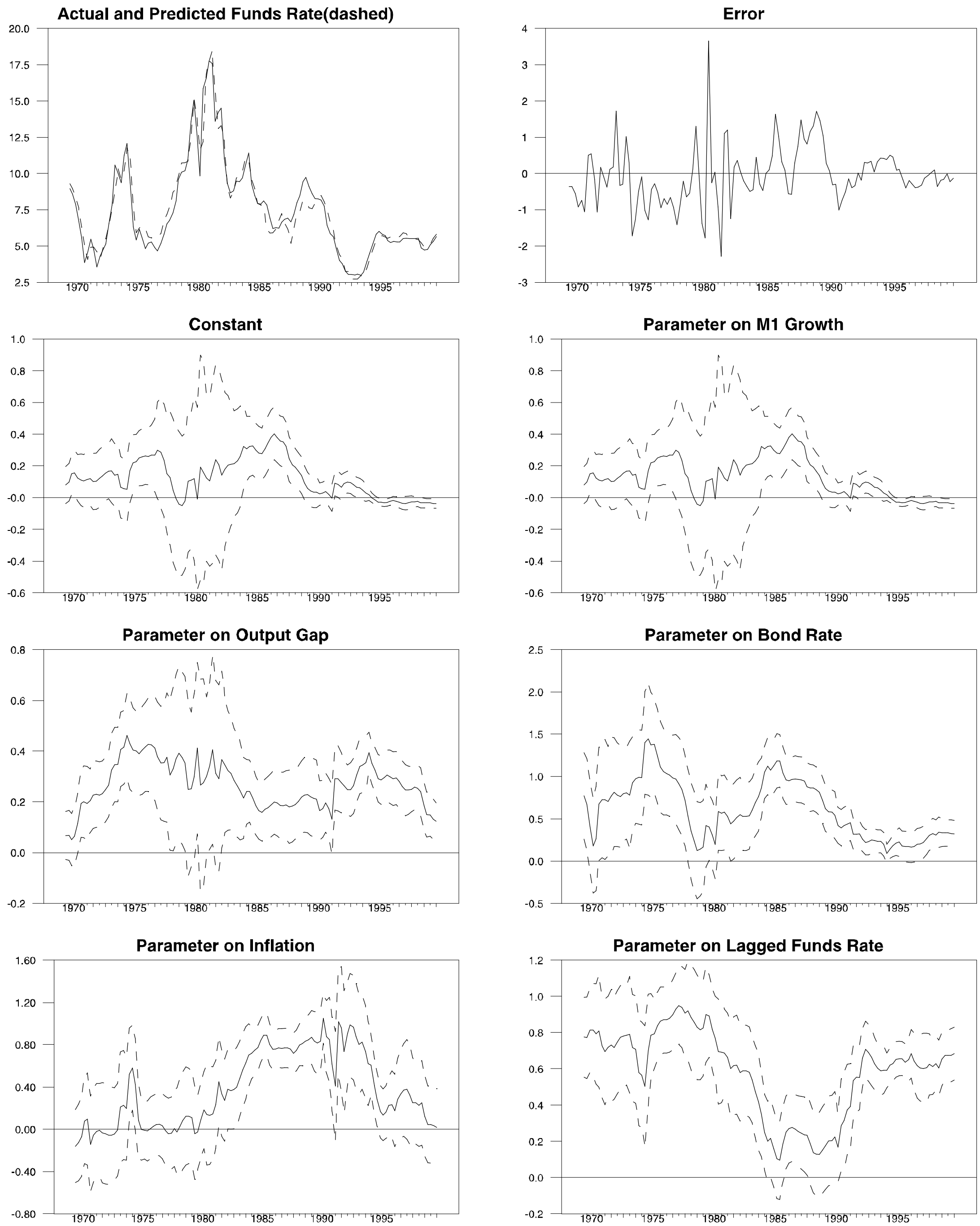

Figure 1. Estimated Taylor-Rule for the U.S. Economy

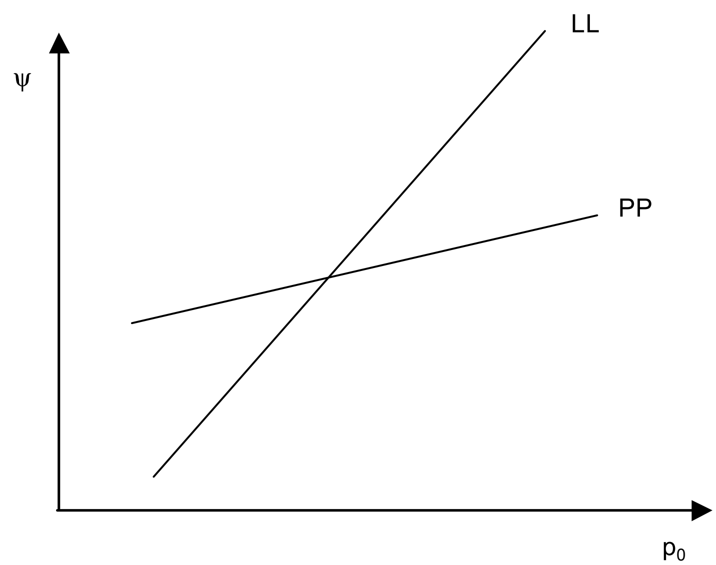

Figure 2. Temporary Equilibrium

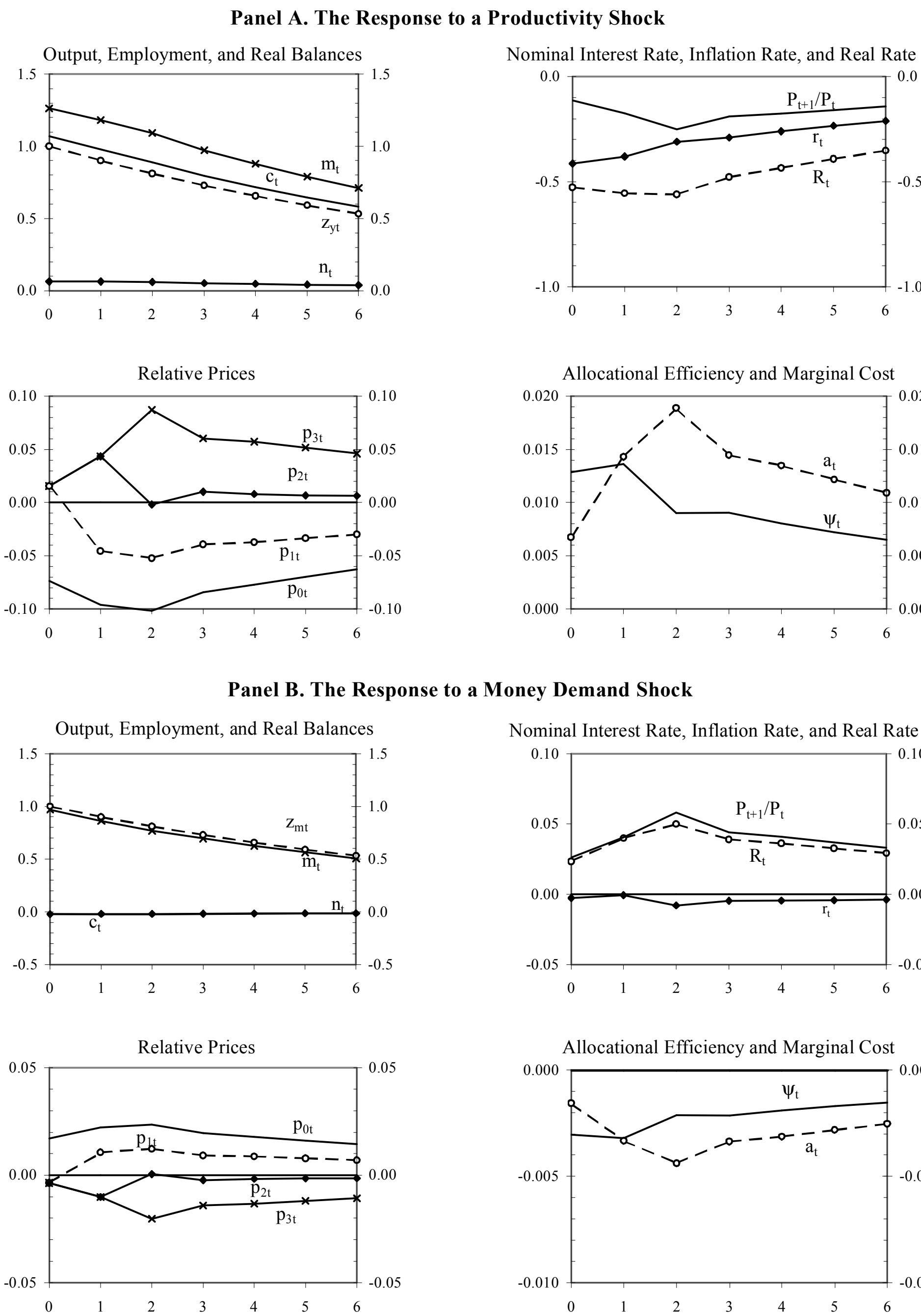

Figure 3. The Full-Information Case

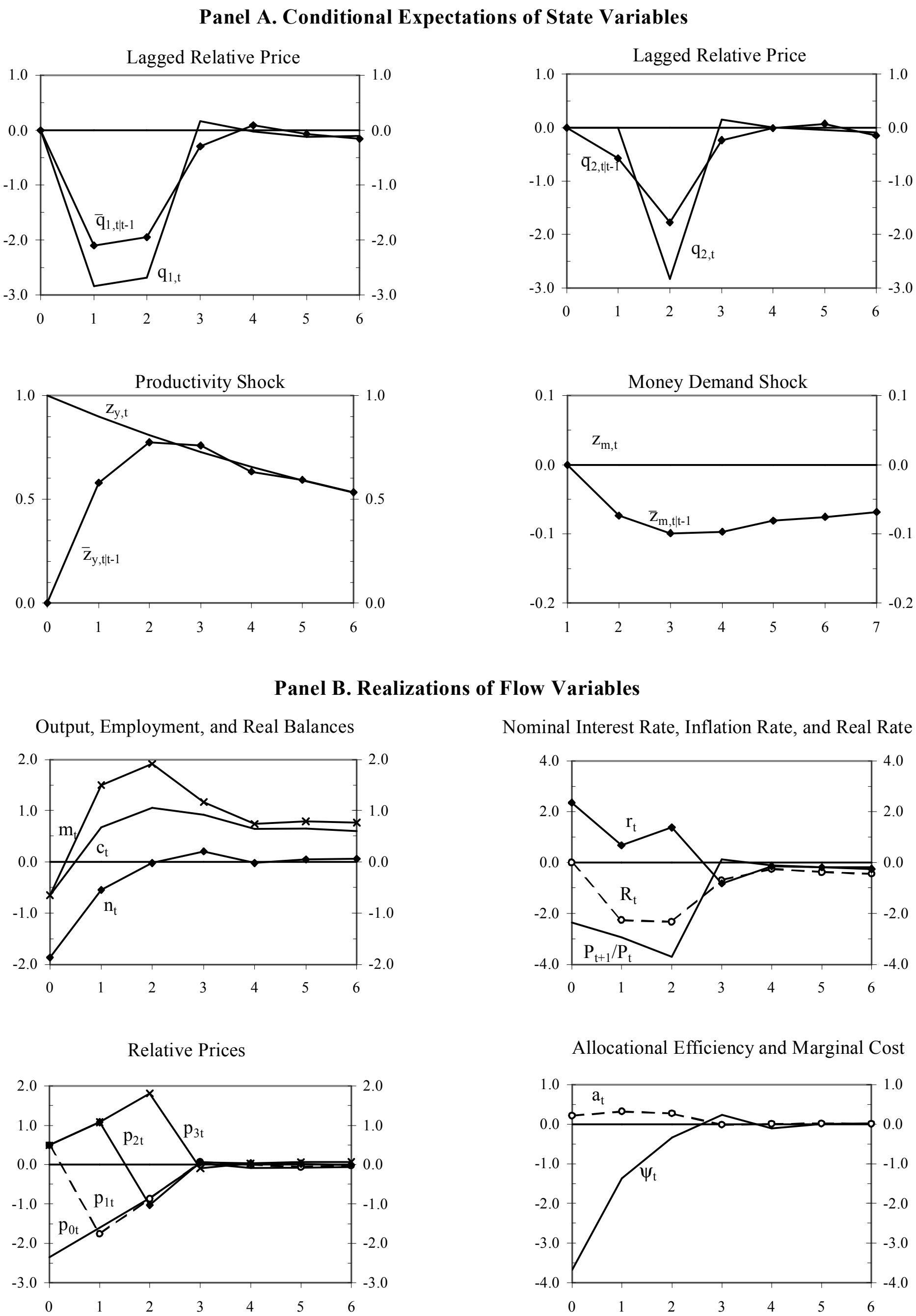

Figure 4. The Response to a Productivity Shock with Information on Lagged Output

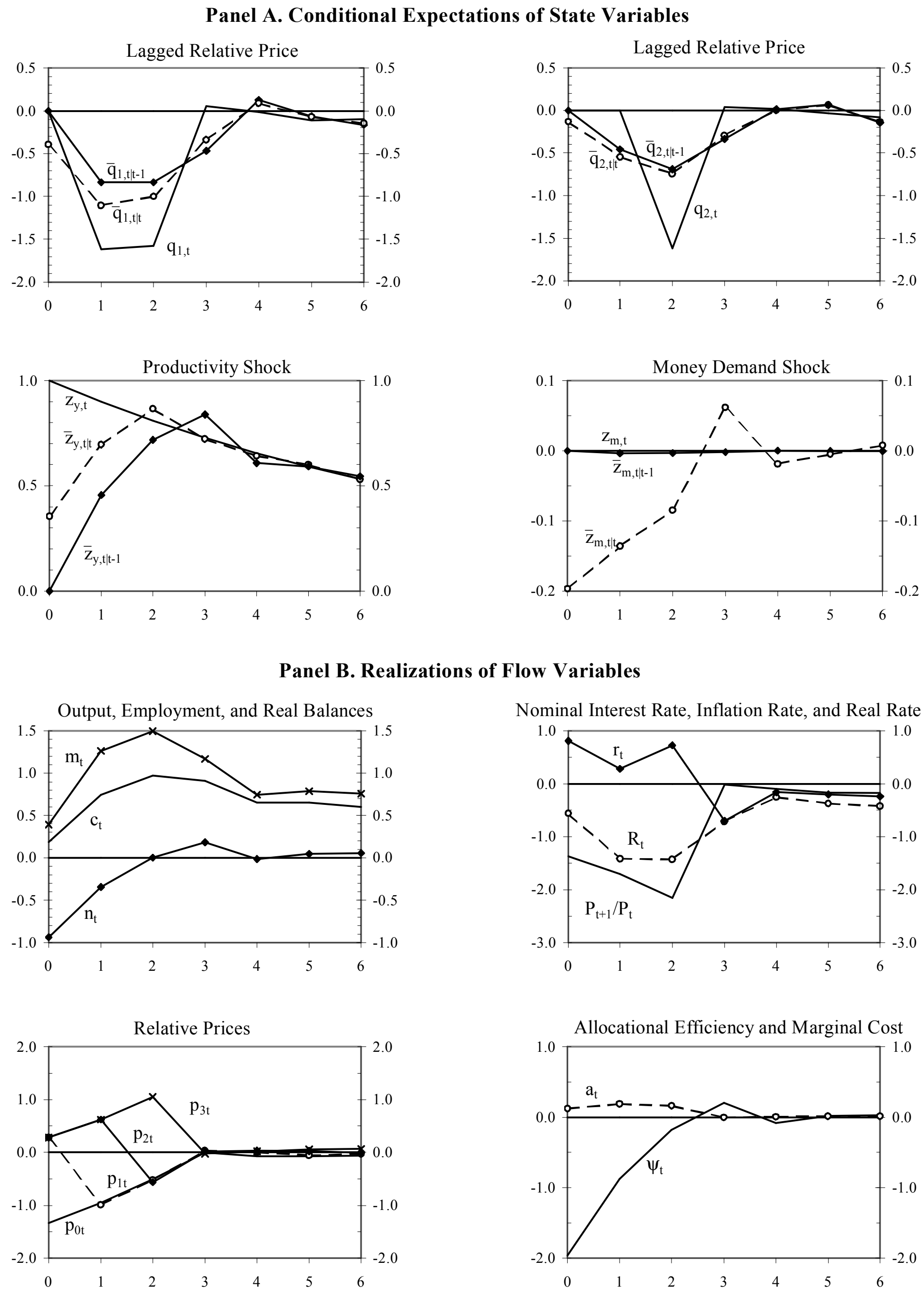

Figure 5. The Response to a Productivity Shock with Information on Contemporaneous Money and Lagged Output

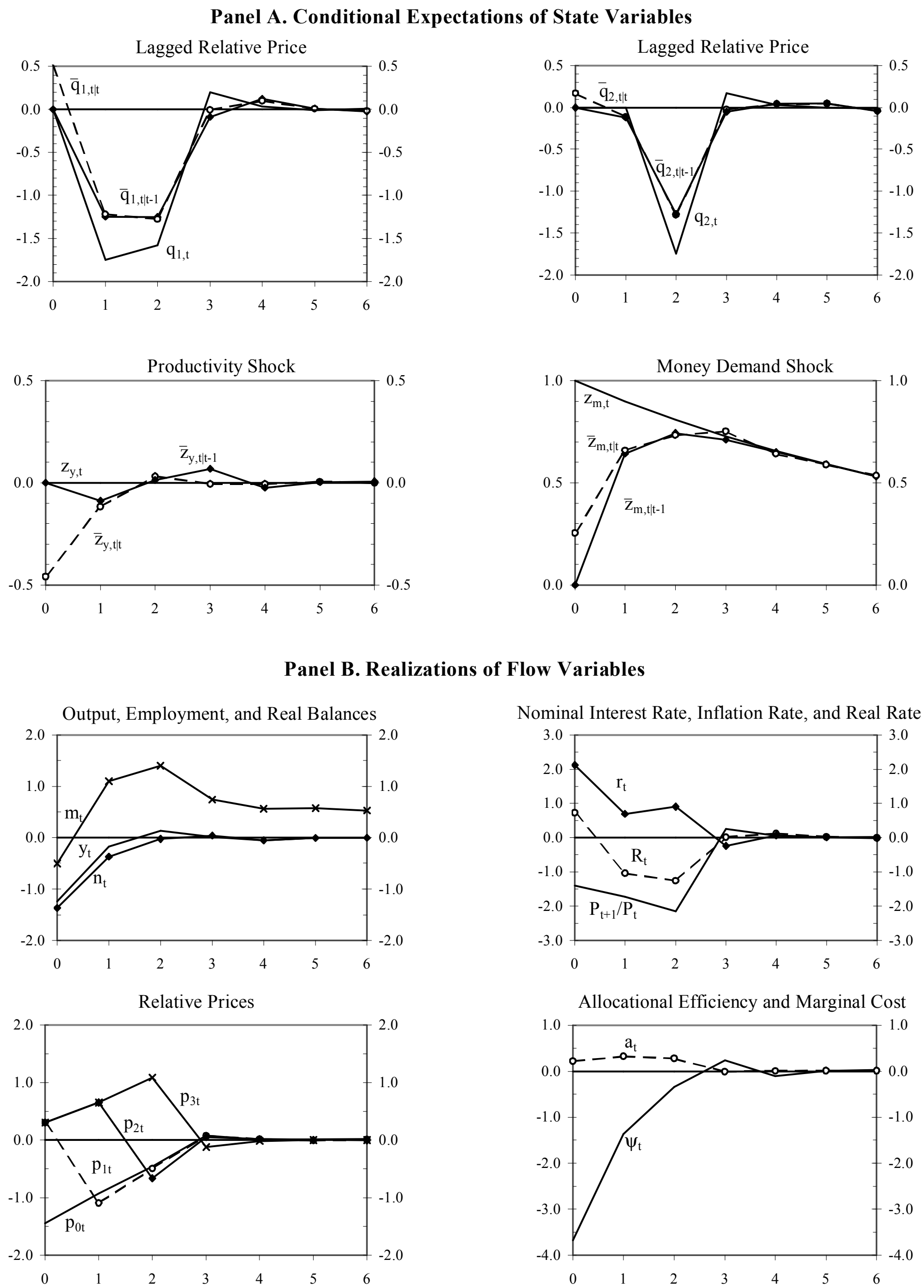

Figure 6. The Response to a Money Demand Shock with Information on Contemporaneous Money and Lagged Output