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Large and Small Sellers:
A Theory of Equilibrium Price Dispersion with Sequential Search[*]

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Abstract

The paper studies equilibrium pricing in a product market for an indivisible good where buyers search for sellers. Buyers search sequentially for sellers but do not meet every seller with the same probability. Specifically, a fraction of the buyers' meetings lead to one particular large seller, while the remaining meetings lead to one of a continuum of small sellers. In this environment, the small sellers would like to set a price that makes the buyers indifferent between purchasing the good and searching for another seller. The large seller would like to price the small sellers out of the market by posting a price that is low enough to induce buyers not to purchase from the small sellers. These incentives give rise to a game of cat and mouse, whose only equilibrium involves mixed strategies for both the large and the small sellers. The fact that the small sellers play mixed strategies implies that there is price dispersion. The fact that the large seller plays mixed strategies implies that prices and allocations vary over time. We show that the fraction of the gains from trade accruing to the buyers is positive and nonmonotonic in the degree of market power of the large seller. As long as the large seller has some positive but incomplete market power, the fraction of the gains from trade accruing to the buyers depends in a natural way on the extent of search frictions.

JEL Codes: D21, D43.

Keywords: Imperfect competition, Search frictions, Price dispersion.

1 Introduction

We propose a novel theory of equilibrium price dispersion in product markets with search frictions. As in Diamond (1971), buyers search for sellers sequentially. In contrast to Diamond (1971), buyers do not meet all sellers with the same probability. Specifically, a fraction of the buyers' meetings leads to one particular large seller, while the remaining meetings leads to one of a continuum of small sellers. We prove that the unique equilibrium of this model is such that sellers post a nondegenerate distribution of prices and buyers capture a positive fraction of the gains from trade. The fraction of gains from trade accruing to the buyers is hump-shaped with respect to the market power of the large seller. However, for any degree of market power of the large seller, the fraction of gains from trade accruing to the buyers converges to one when search frictions vanish, and to zero when search frictions become arbitrarily large.

In a famous paper, Diamond (1971) analyzes a product market where buyers search *sequentially* for sellers. He finds that the equilibrium of this market is such that all sellers post the monopoly price and buyers capture none of the gains from trade. Moreover, he finds that this is the only equilibrium independently of the extent of search frictions. This result, which is popularly known as the *Diamond Paradox*, is problematic for several reasons. From the empirical point of view, the result flies against the evidence documenting the existence of a great deal of price dispersion for identical goods (see, e.g., Sorensen 2000 or Kaplan and Menzio 2014). From the theoretical point of view, the result implies a puzzling discontinuity in outcomes since, without search frictions, all sellers post the competitive price and buyers capture all of the gains from trade. Yet the logic behind the Diamond Paradox is rather strong. Every seller wants to post the buyer's reservation price, i.e., the price that makes a buyer indifferent between purchasing the good and searching for another seller. But if every seller posts the same price, the option value of searching for another seller is zero and the buyer's reservation price must be equal to the monopoly price.

In a series of closely related papers, Butters (1977), Varian (1980), and Burdett and Judd (1983) identify an alternative search process that leads to equilibrium price dispersion and, in turn, to a resolution of the Diamond Paradox. In particular, they consider a search market where some buyers contact multiple sellers *simultaneously*, while other buyers contact one seller at a time. They find that, in equilibrium, sellers post different prices and buyers capture a positive fraction of the gains from trade. The intuition behind

their result is clear. An equilibrium in which all sellers post the monopoly price (or any common price above the cost of production) cannot exist, as an individual seller would have an incentive to post a slightly lower price and trade not only with those buyers who have only contacted him, but also with those buyers who have contacted him, as well as other sellers. This undercutting process cannot lead to an equilibrium in which all sellers post a price equal to the cost of production, as an individual seller can always attain a strictly positive profit by charging a slightly higher price and trading only with buyers who have failed to contact multiple sellers. Hence, the equilibrium must involve a distribution of prices and buyers capture some of the gains from trade.

The assumption that some buyers contact multiple sellers simultaneously is quite strong. The assumption does not mean that there are some buyers that can freely recall previously contacted sellers. Indeed, even if some buyers could freely recall sellers, the only equilibrium would be such that all sellers post the monopoly price and buyers capture none of the gains from trade. The assumption really means that there are some buyers that *come into contact with multiple sellers before being able to decide whether to stop searching.* This observation motivates our paper, which advances a theory of equilibrium price dispersion in markets where search is genuinely sequential, in the sense that buyers have the option to stop searching after meeting any individual seller.

We consider a product market populated by buyers—each demanding one unit of an indivisible good—and sellers—each producing the good at the same cost. At the beginning of each day of trading, sellers post prices. Then buyers observe the price distribution and start searching for sellers. As in Diamond (1971), buyers search for sellers sequentially. However, in contrast to Diamond (1971), buyers do not meet all sellers with the same probability. Specifically, a fraction of the buyers' meetings leads to one particular *large seller*, while the remaining meetings lead to one of a continuum of *small sellers*. We prove that an equilibrium in this market exists and is unique. In equilibrium, the large seller randomizes over his price from a distribution whose support is a closed, convex, non-empty subset of the interval between the sellers' cost and the buyers' valuation of the good. The small sellers post a distribution of prices whose support is also a closed, convex and non-empty subset of the interval between cost and valuation of the good. The buyers capture a positive fraction of the gains from trade, which varies depending on the realization of the large seller's price.

The intuition behind these results is straightforward. The small sellers would like to

set a price that makes the buyers indifferent between purchasing the good and searching for another seller. The large seller would like to price the small sellers out of the market by posting a price that is low enough to induce buyers not to purchase from the small sellers. These incentives give rise to a game of cat and mouse, whose only equilibrium involves mixed strategies for both the large and the small sellers. The fact that the small sellers play mixed strategies implies that there is price dispersion in equilibrium and, hence, the buyers capture a strictly positive fraction of the gains from trade. The fact that the large seller plays mixed strategies implies that prices and allocations vary over time.

The equilibrium outcomes in our model depend critically on two parameters: the market power of the large seller—as measured by the fraction of meetings that involve the large seller—and the extent of search frictions—as measured by the rate at which buyers meet sellers. We find that the competitiveness of the market is nonmonotonic with respect to the market power of the large seller. When the market power of the large seller vanishes, the environment converges to the one studied by Diamond (1971) and the equilibrium outcomes converges to those of a pure monopoly. In particular, the distributions of prices posted by the large and the small sellers converge to the monopoly price and the fraction of the gains from trade accruing to the buyers goes to zero. Similarly, when the large seller has complete market power, the equilibrium outcomes converge to those of a pure monopoly. However, as long as the market power of the large seller is positive but finite, there is price dispersion and buyers capture a positive fraction of the gains from trade. Thus, the Diamond Paradox can be viewed as the limit of a paradoxically nonmonotonic relationship between the market power of the large seller and the extent of competition.

Whenever the market power of the large seller is positive and incomplete, the competitiveness of the market depends in a natural way on the extent of search frictions. When the search frictions become arbitrarily small, the equilibrium outcomes converge to those of a perfectly competitive market. In particular, the distributions of prices posted by the large and the small sellers converge to the competitive price and the fraction of the gains from trade accruing to the buyers converges to one. In contrast, when the search frictions become arbitrarily large, the equilibrium outcomes converge to those of a pure monopoly. Thus, whenever the market power of the large seller is positive and incomplete, the discontinuity in equilibrium outcomes highlighted by Diamond (1971) disappears: When search frictions vanish, the equilibrium becomes competitive.

Our paper adds to the existing theories of equilibrium price dispersion in product markets with search frictions. As mentioned above, Butters (1977), Varian (1982), and Burdett and Judd (1983) generate price dispersion in the context of a model where search is sometimes sequential and sometimes simultaneous. In contrast, we obtain price dispersion in a model where search is genuinely sequential. Albrecht and Axell (1984) obtain price dispersion in a model of sequential search by introducing heterogeneity in buyers' valuations. However, their model is such that the Diamond Paradox holds for the buyers with the lowest valuation. Hence, in the presence of any entry cost, the lowest valuation buyers would stay out of the market and price dispersion would unravel. In contrast, our theory of price dispersion is robust to the introduction of entry costs. Curtis and Wright (2004) and Gaumont, Schindler and Wright (2006) obtain price dispersion in a model of sequential search by introducing heterogeneity in the gains from trade between buyers and sellers. In contrast, our theory of price dispersion does not require such heterogeneity. Finally, Benabou (1989) obtains price dispersion in a model of sequential search by introducing inflation and costs to adjust nominal prices. In contrast, our theory of price dispersion does not rely on nominal rigidities.

There are also several theories that resolve the Diamond Paradox without generating price dispersion. Pissarides (1984) and Mortensen and Pissarides (1994) show that, if prices are determined as the outcome of a bargaining game between buyers and sellers rather than being posted by sellers, then buyers will generally capture a positive fraction of the gains from trade. Montgomery (1991), Shimer (1996), Moen (1997), and Burdett, Shi and Wright (2001) show that, if buyers can direct their search toward particular sellers, then sellers will compete for searchers and buyers will capture a positive fraction of the gains from trade. Carrillo-Tudela, Menzio and Smith (2011) show that, if buyers can recall past sellers and sellers can distinguish between buyers with single and multiple contacts, then buyers extract some of the gains from trade in equilibrium.

2 Environment and definition of equilibrium

In this section, we describe the market for an indivisible good where buyers search sequentially for sellers and face a positive probability of meeting one particular seller. We then state the problem of the buyers, the problem of the large seller, and the problem of the small sellers in this market. Finally, we define an equilibrium.

2.1 Environment

The market for an indivisible good opens daily. The market is populated by a continuum of buyers with measure $b > 0$. Each buyer demands one unit of the good every day.[1] The market is also populated by one large seller and a continuum of small sellers with measure one. Each seller produces the good at the same, constant unit cost.[2]

At the beginning of each day, every seller posts simultaneously and independently a price for the good. We denote as x the price of the large seller and as y the price of a small seller. We also denote as $F(x)$ the probability that the large seller posts a price smaller or equal to x and as $\mathcal{F}$ the support of the price distribution F.[3] Similarly, we denote as $G(y)$ the probability that a small seller posts a price smaller or equal to y and as $\mathcal{G}$ the support of the price distribution G. Both F and G are endogenous objects. We assume that sellers cannot change their price during the day but can freely change their price from one day to the next.[4]

After observing the distribution of posted prices, every buyer enters the market and starts searching for sellers.[5] A searching buyer meets a seller at the Poisson rate λ, where $\lambda > 0$ is a parameter that controls the extent of search frictions in the market. Conditional on meeting a seller, the buyer meets the large seller with probability α and one of the continuum of small sellers with probability $1 - \alpha$, where $\alpha \in (0, 1)$ is a parameter that controls the extent of market power of the large seller. A searching buyer is forced out of the market at the Poisson rate ρ, where $\rho > 0$ is a parameter that controls the cost of searching.

Upon meeting a seller, the buyer decides whether to purchase the good. If the buyer purchases the good at the price p, he enjoys a utility of $u - p$ and exits the market for the day, where $u > 0$ is the buyer's valuation of the good. If the seller sells the good

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[1] Equivalently, one can think that the market is visited by a different group of buyers every day.

[2] Our theory of price dispersion generalizes to the case in which there is more than one large seller, as well as to the case in which all sellers are large. We choose to focus on the case in which there is one large seller and a continuum of small sellers only for the sake of exposition.

[3] Our theory of price dispersion would generalize to the case in which sellers can also adjust their prices during the day, but only every once in a while.

[4] The support of a cumulative distribution function is defined as the smallest closed set whose complement has probability zero.

[5] We assume that buyers observe the distribution of prices in order to side-step issues of learning while searching, which are notoriously difficult and have been tackled only by a handful of papers (see, e.g., Rothschild 1974, and Burdett and Wishwanath 1988). Moreover, the assumption is not unreasonable. For instance, buyers may learn about the price distribution on the Internet before leaving their home in the morning and, later in the day, they may meet a sequence of sellers on the way to and from work.

at the price p, he enjoys a profit of $p - c$ and keeps searching for additional customers, where $c \in [0, u)$ is the seller's cost of producing the good. If the buyer is forced to exit the market before purchasing the good, he enjoys a utility of 0. In order to keep the problem of the buyer stationary, we assume that every day is infinitely long. We also assume that buyers and sellers do not discount their payoffs during the day.

Before defining an equilibrium, a few observations about the environment are in order. First, notice that search is sequential, in the sense that a buyer has the option to purchase the good and stop searching after every meeting with an individual seller. Thus, in contrast to Butters (1977), Varian (1980), and Burdett and Judd (1983), our theory of price dispersion does not hinge on the assumption of simultaneous search. Second, notice that all buyers are identical, in the sense that they have the same valuation of the good. Thus, in contrast to Albrecht and Axell (1984), our theory of price dispersion does not rely on the assumption of buyer heterogeneity. Third, notice that all buyer-seller matches are identical, in the sense that they all involve the same gains from trade. Thus, in contrast to Curtis and Wright (2004) and Gaumont, Schindler and Wright (2006), our theory of price dispersion does not rely on the assumption of heterogeneity in the gains from trade across different buyer-seller meetings. Indeed, the only substantive difference between our environment and the one analyzed by Diamond (1971) is the existence of a large seller, i.e., a seller that is contacted by buyers with positive probability and, hence, whose pricing decision impacts the buyers' value of searching the market.

2.2   Individual problems and definition of equilibrium

We restrict attention to equilibria where the acceptance strategy of the buyers, the pricing strategy of the large seller, $F(x)$, and the pricing strategy of the small sellers, $G(y)$, are all independent of the history of prices and trades in previous days.[6] Given this restriction, the price of the large seller today does not affect the seller's expected profit in the subsequent days. Thus, the large seller chooses today's price so as to maximize today's expected profit. Similarly, the price of a small seller today does not affect the seller's expected profit in the subsequent days. Thus, the small seller chooses today's

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[6]As we shall prove in Section 3, there exists a unique history independent equilibrium. Therefore, if the market opens on a finite number of days, the history independent equilibrium is also the only equilibrium on the last day in which the market is open. By induction, this implies that the history independent equilibrium is the only equilibrium on any previous day as well. We have not ruled out the existence of a non-history independent equilibrium when the market remains open forever.

price so as to maximize today's expected profit. Finally, the buyer's purchasing decision today has no effect on his expected utility in subsequent days. Hence, the buyer chooses to accept or reject a price so as to maximize today's expected utility.

Problem of the buyer. Let $V(x)$ denote today's expected utility for a buyer who is searching, when the price posted by the large seller is x. The buyer's expected utility $V(x)$ is such that

$$\rho V(x) = \lambda \alpha \max\left\{u - x - V(x), 0\right\} + \lambda(1 - \alpha) \int \max\left\{u - y - V(x), 0\right\} dG(y). \quad (1)$$

The left-hand side of (1) is the buyer's annuitized value of searching. The first term on the right-hand side of (1) is the rate at which the buyer meets the large seller, $\lambda\alpha$, times the value of the option of purchasing the good from the large seller, $\max\left\{v - x - V(x), 0\right\}$. The second term on the right-hand side of (1) is the rate at which the buyer meets a small seller, $\lambda(1 - \alpha)$, times the expected value of the option of purchasing the good from a small seller, $\int \max\left\{v - y - V(x), 0\right\} dG(y)$. We shall refer to $V(x)$ as the buyer's *value of searching*.

Let $R(x)$ denote the price that makes the buyer indifferent between purchasing the good and continuing his search, when the price posted by the large seller is x. Formally, $R(x)$ is such that

$$R(x) = u - V(x). \quad (2)$$

We shall refer to $R(x)$ as the buyer's *reservation price*. Clearly, the buyer finds it optimal to purchase the good whenever he meets a seller posting a price strictly smaller than the reservation price. Similarly, the buyer finds it optimal to continue searching whenever he meets a seller posting a price strictly greater than the reservation price. We assume that the buyer purchases the good whenever he meets a seller charging the reservation price.

The buyer's optimal purchasing strategy described above determines the evolution of the population of buyers during the day, as well as the overall number of meetings taking place between buyers and sellers in the day. The measure $b(t, x)$ of buyers who are still searching after t units of time since the beginning of the day is given by

$$b(t, x) = b \exp\left\{-\left[\rho + \lambda\alpha\mathbf{1}[x \leq R(x)] + \lambda(1 - \alpha)G(R(x))\right]t\right\}, \quad (3)$$

where ρ is the rate at which buyers are forced out of the market, $\lambda\alpha\mathbf{1}[x \leq R(x)]$ is the rate

at which buyers meet the large seller and purchase the good[7] and $\lambda(1 - \alpha)G(R(x))$ is the rate at which buyers meet a small seller and purchase the good. Similarly, the measure $m(x)$ of meetings that take place between buyers and sellers throughout the day is given by

$$m(x) = \frac{b\lambda}{\rho + \lambda\alpha\mathbf{1}[x \leq R(x)] + \lambda(1 - \alpha)G(R(x))}. \tag{4}$$

As suggested by the notation, the price x posted by the large seller affects the buyer's value of searching, $V(x)$, the buyer's reservation wage, $R(x)$, the measure of searching buyers, $b(t, x)$, and the overall measure of meetings between buyers and sellers, $m(x)$. Intuitively, when the large seller posts a higher price, buyers have less to gain from searching and, hence, they are willing to purchase the good at higher prices from the small sellers. This implies that, when the large seller posts a higher price, buyers exit the product market more quickly and, hence, they meet fewer sellers.

Problem of the large seller. Let $L(x)$ denote the profit expected by the large seller today if he posts a price of x. The profit $L(x)$ is given by

$$L(x) = m(x)\alpha\mathbf{1}[x \leq R(x)](x - c). \tag{5}$$

The expression in (5) is easy to understand. The seller is contacted by a measure $m(x)\alpha$ of buyers. Each one of these buyers purchases the good with probability $\mathbf{1}[x \leq R(x)]$. Each unit of the good purchased by the buyers gives the seller a profit of $x - c$.

The large seller chooses the price x so as to maximize his expected profit $L(x)$. Thus, the large seller finds it optimal to follow the mixed pricing strategy $F(x)$ if and only if $L(x) = L^*$ for all $x \in \mathcal{F}$ and $L(x) \leq L^*$ for all $x \notin \mathcal{F}$, where $\mathcal{F}$ denotes the support of the price distribution F and L^* denotes the maximum of L^* with respect to x.

Notice that, when the price x is greater than the buyer's reservation price $R(x)$, the profit of the large seller $L(x)$ is always equal to zero. When x is smaller than $R(x)$, the price has two countervailing effects on the profit of the large seller. On the one hand, a higher x increases the profit that the large seller enjoys every time it trades with a buyer. On the other hand, a higher x tends to increase the buyer's reservation price. In turn, this tends to increase the fraction of buyers who purchase the good from the small sellers and, consequently, it reduces the fraction of buyers who contact and purchase from the

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[7]Throughout the paper, we denote as $\mathbf{1}[f(x) \leq 0]$ the function that takes the value 1 if the condition in square bracket is satisfied, and the value 0 otherwise.

large seller. Because of these two countervailing effects of x on $L(x)$, it may be the case that the large seller's profit attains its maximum over a non-degenerate interval of prices and, hence, it may be the case that the optimal strategy of the large seller is mixed.

Problem of the small seller. Let $S(y)$ denote the profit expected by the small seller today if he posts a price of y. The profit $S(y)$ is given by

$$S(y) = \int m(x)(1 - \alpha)\mathbf{1}[y \leq R(x)](y - c)dF(x). \tag{6}$$

The expression in (6) is easy to understand. The price posted by the large seller is drawn from the cumulative distribution function $F(x)$. Conditional on the large seller posting the price x, the small seller is contacted by $m(x)(1 - \alpha)$ buyers. Each one of these buyers purchases the good with probability $\mathbf{1}[y \leq R(x)]$. Each unit of the good purchased by the buyers gives the seller a profit of $y - c$.

The small seller chooses the price y so as to maximize his expected profit $S(y)$. Thus, the small seller finds it optimal to follow the mixed pricing strategy $G(y)$ if and only if $S(y) = S^*$ for all $y \in \mathcal{G}$ and $S(y) \leq S^*$ for all $y \notin \mathcal{G}$, where $\mathcal{G}$ denotes the support of the price distribution G and S^* denotes the maximum of $S(y)$ with respect to y.

Notice that the price y has two countervailing effects on the profit of the small seller $S(y)$. On the one hand, a higher y increases the profit that the small seller enjoys every time it trades with a buyer. On the other hand, a higher y increases the probability that the price posted by the large seller is such that the buyer's reservation price $R(x)$ is lower than y. In turn, this implies that a higher y reduces the expected number of buyers who purchase from the small seller. Because of these two countervailing effects of y on $S(y)$, it may be the case that the profit of the small seller attains its maximum over a nondegenerate interval of prices and, hence, it may be the case that the optimal strategy of the small seller is mixed.

Definition of equilibrium. We are now in the position to define an equilibrium for our frictional product market.

Definition 1. *A history independent equilibrium is a tuple (R, F, G) such that: (i) the reservation price function R is such that $R(x) = v - V(x)$ for all x; (ii) the cumulative distribution function F is such that $L(x) = L^*$ for all $x \in F$ and $L(x) \leq L^*$ for all $x \notin F$; (iii) the cumulative distribution function G is such that $S(y) = S^*$ for all $y \in G$ and $S(y) \leq S^*$ for all $y \notin G$.*

3 Characterization of equilibrium

In this section, we establish the existence and uniqueness of the equilibrium and we characterize its main features. First, we establish some properties of the buyer's reservation price, R, of the large seller's mixing distribution, F, and of the small sellers' price distribution, G, that must hold in any equilibrium. These properties imply that any equilibrium will feature price dispersion—in the sense that small sellers post different prices—and price variation—in the sense that the large seller posts different prices on different days. Second, we show that an equilibrium always exist and is unique. Finally, we analyze the effect on equilibrium outcomes of changes in the degree of market power of the large seller—as measured by the parameter α—and in the extent of search frictions—as measured by the parameter λ. We find that the fraction of the gains from trade accruing to the buyers is nonmonotonic in the degree of market power of the large seller. However, given any degree of market power of the large seller, the fraction of the gains from trade accruing to the buyers depends in the natural way on the extent of search frictions. These findings allow us to better understand the puzzling results in Diamond (1971).

3.1 Equilibrium price dispersion and variation

We begin the characterization of the equilibrium through a series of lemmas. First, we establish some general properties of the buyer's reservation price. Second, we use the properties of the buyer's reservation price in order to establish that all sellers attain strictly positive profits. Third, we use the properties of the buyer's reservation price to locate the boundaries of the support of the mixing distribution of the large seller and of the price distribution of the small sellers. Fourth, we prove that the price distribution of the small sellers does not contain any mass points or gaps. This result implies that *any equilibrium features price dispersion*. Finally, we show that the mixing distribution of the large seller is nondegenerate with a mass point at the top. This result implies that *any equilibrium features time-variation in prices*.

The first lemma uses the equilibrium condition (i) to establish some general properties of the buyers' reservation price R.

Lemma 1: (Reservation price). *Let (R, F, G) be an equilibrium. (i) The reservation price $R(x)$ is continuous and such that $R(x) > c$ and $R(x) \leq u$ for all $x \in [c, u]$; (ii) There exists a unique $x^* \in (c, u]$ such that $R(x^*) = x^*$, $R(x) > x$ for all $x \in [c, x^*)$ and $R(x) < x$*

for all $x \in (x^*, u]$; *(iii) For any* x_1 *and* x_2 *such that* $c \leq x_1 < x_2 \leq x^*$, $R(x_2) - R(x_1) > 0$ *and* $R(x_2) - R(x_1) < x_2 - x_1$. *For any* x_1 *and* x_2 *such that* $x^* \leq x_1 < x_2 \leq u$, $R(x_2) - R(x_1) = 0$.

Proof: In Appendix A.

The properties of the buyer's reservation price are intuitive. First, it is clear that the buyer's reservation price, $R(x)$, is strictly greater than the seller's marginal cost c and smaller than the buyer's valuation u. To see why this is the case, notice that none of the sellers post prices below the marginal cost c, as this would give them strictly negative profits. Therefore, buyers only find sellers posting prices greater or equal to c. Since finding sellers is time consuming, the buyer's value of searching $V(x)$ is strictly smaller than $u - c$ and, in turn, the reservation price is strictly greater than c. Inspection of (1), immediately reveals that the buyer's value of searching is positive and, in turn, the reservation price is smaller than u.

Second, it is clear that the price posted by the buyer's reservation price, $R(x)$, is increasing in the price posted by the large seller, x, and that it is greater than x when x is relatively low and smaller than x when x is relatively high. If the large seller posts a price x equal to the marginal cost c, $R(x)$ is greater than x because $R(x) > c$. If the large seller increases his price x above c, the buyer's value of searching $V(x)$ declines and, consequently, the buyer's reservation price $R(x)$ increases. However, since buyers encounter the large seller only in a fraction of their meetings, $V(x)$ declines less than one-for-one with x and, consequently, $R(x)$ increases less than one-for-one with x. In turn, this implies that there exists a price x^* such that, if x greater than x^*, the buyer's reservation price is lower than the price posted by the large seller. As we shall see, the price x^* plays a key role in the characterization of the equilibrium.

The first part of Lemma 1 states that the buyer's reservation price is always strictly greater than the seller's marginal cost. Given this feature of the buyer's optimal strategy, sellers can always achieve strictly positive profits. This is formally established by the next lemma.

Lemma 2: (Equilibrium profit). *Let* (R, F, G) *be an equilibrium. The maximized profits of the large and small sellers are strictly positive, i.e.* $L^* > 0$ *and* $S^* > 0$.

Proof: To prove $L^* > 0$, it is sufficient to note that the large seller can achieve a strictly positive profit by posting the price x^*. In fact, by posting x^*, the large seller can trade

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with all the buyers it meets—as $R(x^*) = x^*$—and it can attain a strictly positive profit on each trade—as $x^* > c$. Similarly, to prove $S^* > 0$, it is sufficient to note that the small seller can achieve a strictly positive profit by posting the price $R(c)$. In fact, by posting $R(c)$, the small seller can trade with all the buyers it meets—as $R(x) \geq R(c)$ for all x on the support of F—and it can make a strictly positive profit on each trade—as $R(c) > c$. ∎

The next lemma uses the results in Lemma 1 and in Lemma 2 in order to locate the boundaries of the support of the price distributions F and G.

Lemma 3: (Boundaries of $\mathcal{F}$ and $\mathcal{G}$). *Let (R, F, G) be an equilibrium. Denote as $\underline{x}$ and $\overline{x}$ the lowest and the highest price in F. Similarly, denote as $\underline{y}$ and $\overline{y}$ the lowest and the highest price in G. Then: (i) $\overline{x} = x^*$, (ii) $\overline{y} = R(\overline{x})$; (iii) $\underline{y} = R(\underline{x})$.*

Proof: In Appendix B.

The first part of Lemma 3 states that the highest price posted by the large seller, $\overline{x}$, must be equal to x^*. To see why this is the case, it is sufficient to show that, if $\overline{x}$ were different from x^*, the profit of the large seller would not be maximized at $\overline{x}$ and, hence, part (ii) in the definition of equilibrium would be violated.

First, suppose $\overline{x} > x^*$. In this case, the large seller attains a profit of zero by posting the price $\overline{x}$ because $R(\overline{x}) < \overline{x}$. In contrast, the large seller can attain a strictly positive profit by posting the price x^* because $R(x^*) = x^*$ and $x^* > c$. Thus, $\overline{x} \leq x^*$. Now, suppose $\overline{x} < x^*$. Notice that a small seller never posts a price y strictly greater than $R(\overline{x})$, as doing so means being priced out of the market by the large seller with probability one and, consequently, making no profit. Hence, the distribution of prices among small sellers is such that $G(R(\overline{x})) = 1$. In turn, $G(R(\overline{x})) = 1$ implies that the large seller does not maximize his profit by posting the price $\overline{x}$. Indeed, by posting the price x^* rather than the price $\overline{x}$, the large seller makes the same number of trades as both prices are below the buyers' reservation price—$R(x^*) \leq x^*$ and $R(\overline{x}) < \overline{x}$—and at both prices none of the small sellers is priced out of the market—$G(R(x^*)) = G(R(\overline{x})) = 1$. On the other hand, by posting the price x^* rather than the price $\overline{x}$, the large seller enjoys a higher profit per trade. Thus $\overline{x} \geq x^*$. Overall, any equilibrium must be such that $\overline{x} = x^*$.

The second part of Lemma 3 states the highest price posted by the small seller, $\overline{y}$, must be equal to $R(\overline{x})$. To see why this is the case, it is sufficient to show that, if $\overline{y}$ were

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different from $R(\overline{x})$, the profit of the small seller would not be maximized at $\overline{y}$ and, hence, part (iii) in the definition of equilibrium would be violated.

We have already argued that a small seller never posts a price strictly greater than $R(\overline{x})$. Thus, $\overline{y} \leq R(\overline{x})$. Now, suppose $\overline{y} < R(\overline{x})$. If this is the case, the profit of the large seller is strictly increasing in x over the interval between $R^{-1}(\overline{y})$ and $\overline{x}$. For any price in the interval $[R^{-1}(\overline{y}), \overline{x}]$, the large seller makes the same number of trades as any such price is below the buyers' reservation price—i.e., $R(x) \leq x$—and at any such price none of the small sellers is priced out of the market—$G(R(x)) = 1$. However, for higher prices in the interval $[R^{-1}(\overline{y}), \overline{x}]$, the large seller enjoys a higher profit per trade. This implies that the large seller does not post any price $x \in [R^{-1}(\overline{y}), \overline{x})$, i.e. $F(R^{-1}(\overline{y})-) = F(\overline{x}-)$ [8]. In turn, the fact that $F(R^{-1}(\overline{y})-) = F(\overline{x}-)$ implies that the small seller can attain a higher profit by posting the price $R(\overline{x})$ rather than the price $\overline{y}$. In fact, by posting $R(\overline{x})$ rather than $\overline{y}$, the small seller makes the same number of trades, as both prices are lower than the buyers' reservation price with probability $F(R^{-1}(\overline{y})-)$. However, by posting $R(\overline{x})$ rather than $\overline{y}$, the small seller enjoys a higher profit per trade. Thus, $\overline{y} \geq R(\overline{x})$. Overall, we have shown that any equilibrium must be such that $\overline{y} = R(\overline{x})$.

The last part of Lemma 3 states that the lowest price posted by the seller, $\underline{y}$, must be equal to $R(\underline{x})$. The proof of this result is also intuitive. First, suppose $\underline{y} < R(\underline{x})$. In this case, the small seller can attain a higher profit by posting the price $R(\underline{x})$ rather than $\underline{y}$. By posting $R(\underline{x})$ rather than $\underline{y}$, the small seller makes the same number of trades as both prices are below the buyers' reservation price with probability one. However, by posting $R(\underline{x})$ rather than $\underline{y}$, the small seller would enjoy a higher profit per trade. Therefore, the equilibrium condition (iii) is violated. Now, suppose $\underline{y} > R(\underline{x})$. In this case, the large seller could attain a higher profit by posting the price $R^{-1}(\underline{y}) - \epsilon$ rather than the price $\underline{x}$, for some small $\epsilon > 0$. By posting either price, the large seller prices out of the market all the small sellers and, hence, enjoys the same number of trades. However, by posting $R^{-1}(\underline{y}) - \epsilon$, the large seller enjoys a larger profit per trade. Therefore, the equilibrium condition (ii) is violated. Overall, we have established that the equilibrium conditions (ii) and (iii) can only be fulfilled if $\underline{y} = R(\underline{x})$.

The next lemma establishes some additional properties of the distribution of prices among the small sellers.

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[8] Throughout the paper, we denote $\lim_{x \to x_0^-} f(x)$ as $f(x-)$ and $\lim_{x \to x_0^+} f(x)$ as $f(x+)$.

Lemma 4: (Properties of G) *Let (R, F, G) be an equilibrium. (i) The support of the distribution G is some interval $[\underline{y}, \overline{y}]$, with $c < \underline{y} < \overline{y} \le u$. (ii) The distribution G has no mass points.*

Proof: In Appendix C.

The first part of Lemma 4 states that there are no gaps in the price distribution of the small sellers. This intuition behind this result is simple. Suppose that $G(y_1) = G(y_2-)$ for some $y_1, y_2 \in \mathcal{G}$ and $y_1 < y_2$. If this is the case, the profit of the large seller is strictly increasing in x over the interval $[R^{-1}(y_1), R^{-1}(y_2))$. For any price $x \in [R^{-1}(y_1), R^{-1}(y_2))$, the large seller makes the same number of trades, as it prices out of the market a fraction of $1 - G(y_1)$ of small sellers. However, for higher prices in $[R^{-1}(y_1), R^{-1}(y_2))$, the large seller enjoys a higher profit per trade. Therefore, the large seller does not find it optimal to post any price $x \in [R^{-1}(y_1), R^{-1}(y_2))$, i.e. $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$. In turn, the fact that $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$ implies that the small seller can attain a higher profit by posting the price y_2 rather than the price y_1. By posting y_2 rather than y_1, the small seller makes the same number of trades as, for both prices, the probability of being priced out of the market is equal to $F(R^{-1}(y_1)-)$. However, by posting y_2 rather than y_1, the small seller enjoys a higher profit per trade. Therefore, y_1 cannot be on the support of the distribution G.

The second part of Lemma 4 states that there are no mass points in the price distribution of the small sellers. To see why, suppose there is a mass point at some price $y_1 < \overline{y}$. If this is the case, the profit of the large seller is such that $L(R^{-1}(y_1)-) > L(R^{-1}(y_1))$. Indeed, the large seller makes a discretely larger number of trades at a price x infinitesimally smaller than $R^{-1}(y_1)$, as this price cuts out of the market the mass of small sellers posting the price y_1. The fact that $L(R^{-1}(y_1)-) > L(R^{-1}(y_1))$ implies the large seller does not find it optimal to post any price in some interval $[R^{-1}(y_1), R^{-1}(y_2)]$, i.e. $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$. In turn, the fact that $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$ implies that the profit of the small seller is strictly increasing in y over the interval $[y_1, y_2)$. Indeed, for any $y \in [y_1, y_2)$, the small seller makes the same number of trades as the probability of being priced out of the market is always equal to $F(R^{-1}(y_1)-)$. However, for higher y's in $[y_1, y_2)$, the small seller enjoys a higher profit per trade. Therefore, the profit of the small seller does not attain its maximum at the price y_1 and, consequently, the distribution G cannot have a mass point at $y_1 < \overline{y}$. A similar argument can then be used to rule out the possibility of a mass point at $y_1 = \overline{y}$.

Having ruled out the existence of gaps and mass points, we can conclude that the support of the price distribution G must be some nondegenerate interval $[\underline{y}, \overline{y}]$. Moreover, notice that the lowest price on the distribution, $\underline{y}$, must be strictly greater than the seller's marginal cost c, as $\underline{y} = R(\underline{x})$, $\underline{x} \geq c$ and $R(x) > c$ for all $x \geq c$. Similarly, notice that the highest price on the distribution, $\overline{y}$, must be smaller than the buyer's valuation u, as $\overline{y} = R(\overline{x})$, $R(\overline{x}) = x^*$ and $x^* \leq u$.

The next lemma establishes some additional properties of the distribution from which the large seller draws his price.

Lemma 5: (Properties of F) *Let (R, F, G) be an equilibrium. (i) The support of the distribution F is some interval $[\underline{x}, \overline{x}]$, with $c < \underline{x} < \overline{x} \leq u$. (ii) The distribution F has one and only one mass point at the price $\overline{x}$.*

Proof: In Appendix D.

The first part of Lemma 5 states that the distribution F has no gaps. The second part of Lemma 5 states that the distribution F has one (and only one) mass point at the price $\overline{x}$. It is easy to understand why this is the case. First, suppose F does not have a mass point at $\overline{x}$. In this case, the profit of the small seller does not attain its maximized value at the price $\overline{y}$ and, hence, the equilibrium condition (iii) is violated. In fact, if the small seller posts the price $\overline{y}$, he is priced out of the market with probability one and, hence, it attains a profit of zero. In contrast, if the small seller posts the price $R(c)$, he can attain a strictly positive profit. Next, suppose that F has a mass point at some price $x_1 < \overline{x}$. In this case, the profit of the small seller is discontinuous at $R(x_1)$. Specifically, $S(R(x_1)+) < S(R(x_1))$ since the small seller would make a discretely larger number of sales by posting a price equal to $R(x_1)$ rather than by posting any price strictly greater than $R(x_1)$. In turn, $S(R(x_1)+) < S(R(x_1))$ implies that the small seller never posts a price in some interval $(R(x_1), R(x_2))$. However, this is not consistent with equilibrium because the distribution of prices among small sellers cannot have gaps.

The above observations imply that the support of the distribution F is some interval $[\underline{x}, \overline{x}]$. The lower bound of the support, $\underline{x}$, is strictly greater than the sellers' marginal cost c because of search frictions. The upper bound of the support, $\overline{x}$, is strictly smaller than the buyers' valuation u because of price dispersion. Moreover, the upper bound is strictly greater than the lower bound because, as established in Lemma 3, $\overline{x} = R^{-1}(\overline{y})$ and $\underline{x} = R^{-1}(\underline{y})$ and, as established in Lemma 4, $\overline{y}$ is strictly greater than $\underline{y}$. Therefore, the distribution from which the large seller draws his price is nondegenerate.

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The results in Lemmas 4 and 5 already offer a rather sharp characterization of equilibrium. Lemma 4 guarantees that, in any equilibrium, there will be variation in the prices posted by different sellers for the same good in the same period of time and, hence, the Diamond paradox will not hold. Lemma 5 guarantees that, in any equilibrium, there will be variation over time in the large seller's price and, consequently, in the buyers' reservation price, in the number of transactions, and in the distribution of prices across different transactions. We have thus established the following theorem.

Theorem 1: (Price dispersion and price variation). *In any equilibrium, the distribution of prices among sellers is nondegenerate and varies over time.*

The intuition behind Theorem 1 is straightforward. The small sellers would like to set a price that makes the buyers indifferent between purchasing the good and searching for another seller. The large seller would like to price the small sellers out of the market by posting a price that is low enough to induce buyers not to purchase from the small sellers. These incentives give rise to a game of cat and mouse, whose only equilibrium involves mixed strategies for both the large and the small sellers. The fact that the small sellers play mixed strategies implies that there is price dispersion in equilibrium. The fact that the large seller plays mixed strategies implies that prices and allocations vary over time.

3.2   Existence and uniqueness of equilibrium

Using the equilibrium conditions in Definition 1 and Lemmas 1 through 5, we can solve for the reservation price of buyers, R, the distribution of prices for the large seller, F, and the distribution of prices for the small sellers, G.

The equilibrium condition (ii) states that the profit of the large seller is maximized at every price on the support of the distribution F. That is, $L(x) = L^*$ for all $x \in \mathcal{F}$. Lemma 5 establishes that the support of the distribution F is some interval $[\underline{x}, \overline{x}]$. That is, $\mathcal{F} = [\underline{x}, \overline{x}]$. Therefore, for all $x \in [\underline{x}, \overline{x}]$, we have

$$
\begin{aligned}
L^* \quad &= L(x) \\
&= \frac{b\lambda\alpha(x - c)}{\rho + \lambda\alpha + \lambda(1 - \alpha)G(R(x))}.
\end{aligned} \tag{7}
$$

Equation (7) pins down the maximized profit of the large seller. In fact, Lemma 3 establishes that the highest price posted by the large seller is x^*, i.e. $\overline{x} = x^*$. Also,

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Lemma 4 establishes that, when the large seller posts the price $\bar{x}$, none of the small sellers is priced out of the market, i.e. $G(R(\bar{x})) = 1$. Using these facts, we can solve for the equation $L^* = L(\bar{x})$ with respect to L^* and obtain

$$L^* = \frac{b\lambda\alpha(x^* - c)}{\rho + \lambda}. \tag{8}$$

Having solved for the maximized profit of the large seller, we can use equation (7) to pin down the distribution of prices among small sellers. Intuitively, equation (7) states that—as the price of the large seller falls from $\bar{x}$ to $\underline{x}$—the decline in the profit that he enjoys on each trade must be exactly compensated by an increase in the number of trades that he makes. And since the increase in the number of trades made by the large seller depends on the distribution of small sellers, equation (7) pins down the distribution of prices among small sellers. Formally, using (8), we can solve (7) with respect to $G(R(x))$ and obtain

$$G(R(x)) = 1 - \frac{(\rho + \lambda)(x^* - x)}{\lambda(1 - \alpha)(x - c)}. \tag{9}$$

Having solved for the distribution of prices among the small sellers, we can solve for the lowest price posted by the large seller. In fact, Lemma 4 establishes that, when the large seller posts the price $\underline{x}$, none of the small sellers is priced out of the market, i.e. $G(R(\underline{x})) = 0$. Using this fact, we can solve equation (9) with respect to $\underline{x}$ and obtain

$$\underline{x} = c + \frac{\rho + \lambda\alpha}{\rho + \lambda}(x^* - c). \tag{10}$$

The equilibrium condition (iii) states that the profit of the small seller is maximized at every price on the support of the distribution G. That is, $S(y) = S^*$ for all $y \in \mathcal{G}$. Lemma 4 establishes that the support of the distribution G is the interval $[\underline{y}, \bar{y}]$. That is, $\mathcal{G} = [\underline{y}, \bar{y}]$. Hence, for all $y \in [\underline{y}, \bar{y}]$, we have

$$\begin{aligned} S^* &= S(y) \\ &= b\lambda(1 - \alpha)(y - c)\left\{\frac{\mu(\bar{x})}{\rho + \lambda} + \int_{R^{-1}(y)}^{\bar{x}} \frac{F'(x)}{\rho + \lambda\alpha + \lambda(1 - \alpha)G(R(x))}dx\right\}, \end{aligned} \tag{11}$$

where $\mu(\bar{x})$ denotes the probability that the large seller posts the price $\bar{x}$.

Equation (11) can be used to pin down the mixing distribution of the large seller. Intuitively, equation (11) states that—as the price of the small seller falls from $\bar{y}$ to $\underline{y}$—

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the increase in the profit that he enjoys on each trade must be exactly compensated by an increase in the expected number of trades that he makes. And since the number of trades made by the small seller depends on the probability that the large seller prices him out of the market, equation (11) pins down the mixing distribution of the large seller.

Formally, equation (11) implies that $S'(y) = 0$ for all $y \in [\underline{y}, \overline{y}]$. Since, Lemma 4 establishes that $\underline{y} = R(\underline{x})$ and $\overline{y} = R(\overline{x})$, it follows that $S'(R(x)) = 0$ for all $x \in [\underline{x}, \overline{x}]$ or, equivalently,

$$\phi'(x) = -\phi(x)\frac{R'(x)}{R(x) - c}, \tag{12}$$

where $\phi(x)$ is defined as

$$\phi(x) = \frac{\mu(\overline{x})}{\rho + \lambda} + \int_x^{\overline{x}} \frac{F'(s)}{\rho + \lambda\alpha + \lambda(1 - \alpha)G(R(s))} ds. \tag{13}$$

The differential equation in (12) can be solved for $\phi(x)$ to obtain

$$\phi(x) = \phi(\overline{x})\frac{R(\overline{x}) - c}{R(x) - c}. \tag{14}$$

Using the solution for $\phi(x)$ in (14) and the definition of $\phi(x)$ in (13), we can recover the mixing distribution F. In particular, after equating the right-hand side of (14) to the right-hand side of (13) and differentiating with respect to x, we find that the derivative of F is given by

$$F'(x) = \phi(\overline{x})(\rho + \lambda)\frac{R'(x)(x - c)}{(R(x) - c)^2}. \tag{15}$$

Similarly, after equating the right-hand side of (14) to the right-hand side of (13) evaluated at $x = \overline{x}$, we find that the mass point $\mu(\overline{x})$ is given by

$$\mu(\overline{x}) = \phi(\overline{x})(\rho + \lambda). \tag{16}$$

Since the mixing distribution F must integrate up to one, $\phi(\overline{x})$ must be given by

$$\phi(\overline{x}) = \left\{(\rho + \lambda)\left[1 + \int_{\underline{x}}^{\overline{x}} \frac{R'(x)(x - c)}{(R(x) - c)^2}\right]\right\}^{-1}. \tag{17}$$

The equilibrium condition (i) can be used to solve for the buyer's reservation price. In fact, the condition states that the buyer's reservation price, $R(x)$, equals the difference between the buyer's valuation of the good, u, and the buyer's value of searching, $V(x)$. Using $R(x) = u - V(x)$ and the fact—implied by Lemma 1 and Lemma 3—that $x \leq R(x)$

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for all $x \in [\underline{x}, \overline{x}]$, we can solve the Bellman Equation (1) with respect to $V(x)$. We find that, for all x in the interval $[\underline{x}, \overline{x}]$, the buyer's value of searching is given by

$$V(x) = \frac{\lambda\alpha(u - x) + \lambda(1 - \alpha)\int_{\underline{y}}^{R(x)}(u - y)dG(y)}{\rho + \lambda\alpha + \lambda(1 - \alpha)G(R(x))}. \tag{18}$$

Differentiating (18) with respect to x and using the fact that $V'(x) = -R'(x)$, we find that, for all x in the interval $[\underline{x}, \overline{x}]$, the derivative of the buyer's reservation price is given by

$$R'(x) = \frac{\lambda\alpha(x^* - c)}{(\rho + \lambda)(x - c)}. \tag{19}$$

Then, using the expression for $R'(x)$ in (19) and the fact that $R(x^*) = x^*$, we find that, for all x in the interval $[\underline{x}, \overline{x}]$, the buyer's reservation price is given by

$$R(x) = x^* - \int_{x}^{x^*} R'(s)ds. \tag{20}$$

Finally, we solve for the equilibrium value of x^*. Since $x^* = R(x^*)$ and $R(x^*) = u - V(x^*)$, it follows that $x^* = u - V(x^*)$ or, equivalently,

$$x^* = u - \frac{1}{\rho + \lambda}\left[\lambda\alpha(u - x^*) + \lambda(1 - \alpha)\int_{\underline{y}}^{R(x^*)}(u - y)dG(y)\right]. \tag{21}$$

The integral on the right hand side of (21) is such that

$$\int_{\underline{y}}^{R(x^*)}(u - y)dG(y)$$

$$= u - \overline{y} + \int_{\underline{y}}^{R(x^*)}G(y)dy = u - x^* + \int_{\underline{x}}^{x^*}G(R(x))R'(x)dx \tag{22}$$

$$= u - x^* + \frac{\alpha}{1 - \alpha}(x^* - \underline{x}) + \left(\frac{\lambda\alpha}{\rho + \lambda} - \frac{\alpha}{1 - \alpha}\right)(x^* - c)\log\left(\frac{x^* - c}{\underline{x} - c}\right),$$

where the first equality follows from integration by parts, the second equality follows from $\underline{y} = R(\underline{x})$, $\overline{y} = R(\overline{x}) = R(x^*) = x^*$ and from a change of variable, while the last equality follows by substituting $G(R(x))$ with its equilibrium value in (10), by substituting $R'(x)$ with its equilibrium value in (19), and by solving the integral.

After substituting (22) into (21) and solving for x^*, we obtain

$$x^* = \frac{\rho}{\rho + \psi}u + \frac{\psi}{\rho + \psi}c, \tag{23}$$

where ψ is defined as

$$\psi = \lambda\alpha\frac{\lambda\alpha + \rho}{\rho + \lambda}\left[\frac{\lambda(1-\alpha)}{\rho + \lambda\alpha} - \log\left(\frac{\rho + \lambda}{\rho + \lambda\alpha}\right)\right]. \tag{24}$$

We have now identified a unique candidate equilibrium. The price x^* is uniquely determined by (23). Given x^*, the buyer's reservation price $R(x)$ is uniquely determined by (20). Given x^* and $R(x)$, the lowest price on the support of the distribution F, $\underline{x}$, is uniquely determined by (10), the highest bound on the support of the distribution F, $\overline{x}$, is uniquely determined by $\overline{x} = x^*$, and the shape of the distribution F is uniquely determined by (15) and (16). Similarly, given x^*, $R(x)$ and $\underline{x}$, the lowest price on the distribution G, $\underline{y}$, is uniquely determined by $\underline{y} = R(\underline{x})$, the highest price on the distribution G, $\overline{y}$, is uniquely determined by $\overline{y} = R(\overline{x})$, and the shape of the distribution G is uniquely determined by (9). Hence, there is a unique candidate equilibrium.

The candidate equilibrium described above is indeed an equilibrium. First, F is a proper cumulative distribution function. In fact, the coefficient ψ in (23) is strictly positive and, hence, $x^* > c$, which in turn implies $R(x) - c > 0$ and $R'(x) > 0$ for all $x \in [\underline{x}, \overline{x}]$. Since $R(x) - c > 0$ for all $x \in [\underline{x}, \overline{x}]$, $F'(x)$ is strictly positive for all $x \in [\underline{x}, \overline{x}]$. Moreover, it is immediate to verify that $F(\underline{x}) = 0$ and $F(\overline{x}) = 1$. Second, G is a proper cumulative distribution function. Since $R'(x) > 0$ for all $x \in [\underline{x}, \overline{x}]$, $G'(y)$ is strictly positive for all $y \in [\underline{y}, \overline{y}]$. Moreover, it is immediate to verify that $G(\underline{y}) = 0$ and $G(\overline{y}) = 1$. Third, the candidate equilibrium satisfies all the equilibrium conditions in Definition 1. The candidate equilibrium satisfies condition (ii). Indeed, from the construction of G, it follows that the profit of the large seller takes the same value L^* for all prices on the interval $[\underline{x}, \overline{x}]$. Moreover, it is straightforward to verify that the profit of the large seller is smaller than L^* for all prices outside of the interval $[\underline{x}, \overline{x}]$. The candidate equilibrium satisfies condition (iii). Indeed, from the construction of F, it follows that the profit of the small seller takes the same value S^* for all prices on the interval $[\underline{y}, \overline{y}]$. Moreover, it is straightforward to verify that the profit of the small seller is smaller than S^* for all prices outside of the interval $[\underline{y}, \overline{y}]$. Finally, from the construction of $R(x)$ and x^*, it follows that the candidate equilibrium satisfies condition (i).

The above observations complete the proof of the following theorem.

Theorem 2: (Existence and Uniqueness). *An equilibrium exists and is unique.*
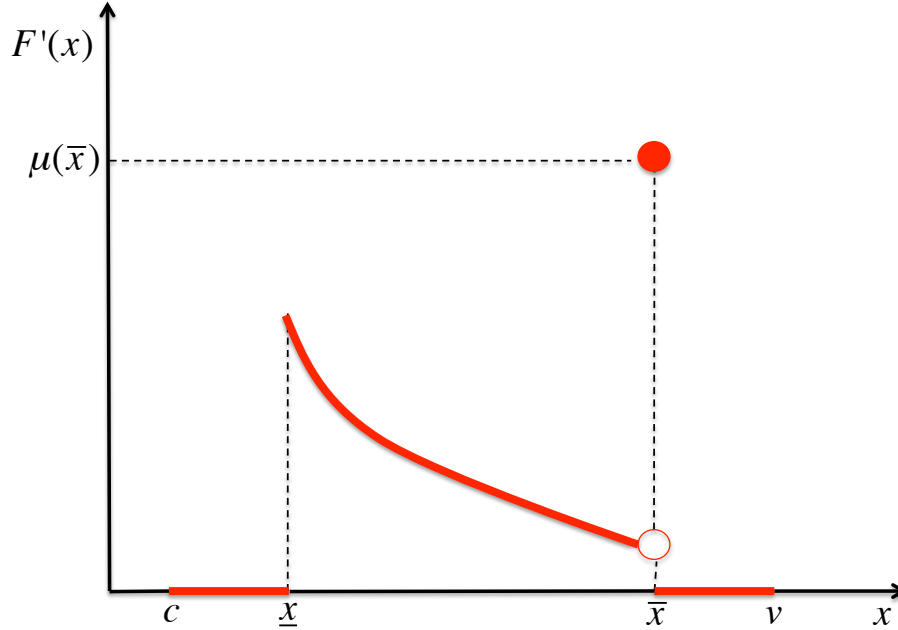
Figure 1: Large seller's price distribution

3.3 Properties of equilibrium and comparative statics

It is useful to summarize the features of the equilibrium. The large seller draws its price from the distribution F, whose properties are illustrated in Figure 1. The support of the distribution is the interval $[\underline{x}, \overline{x}]$, where $\overline{x} = x^*$ is a weighted average of the seller's marginal cost c and the buyer's valuation u, while $\underline{x}$ is strictly greater than c and strictly smaller than $\overline{x}$. The distribution has a mass point of measure $\mu(\overline{x})$ at $\overline{x}$. Moreover, using (15), it is straightforward to verify that the distribution has a strictly positive and strictly decreasing density over the interval $[\underline{x}, \overline{x})$.

The distribution of prices among small sellers is G, whose properties are illustrated in Figure 2. The support of the distribution is the interval $[\underline{y}, \overline{y}]$, where $\overline{y}$ is given by $R(\overline{x})$ and $\underline{y}$ is given by $R(\underline{x})$. Using (20), it is immediate to see that $\overline{y}$ is equal to $\overline{x}$ and $\underline{y}$ is strictly greater than $\underline{x}$ and strictly smaller than $\overline{x}$. Moreover, using (9) and (19), one can show that the distribution G has a strictly positive and strictly increasing density over the interval $[\underline{y}, \overline{y}]$.

The properties of the reservation price of buyers, R, are illustrated in Figure 3. From Lemma 1 and Lemma 3, it follows that the reservation price is strictly greater than the
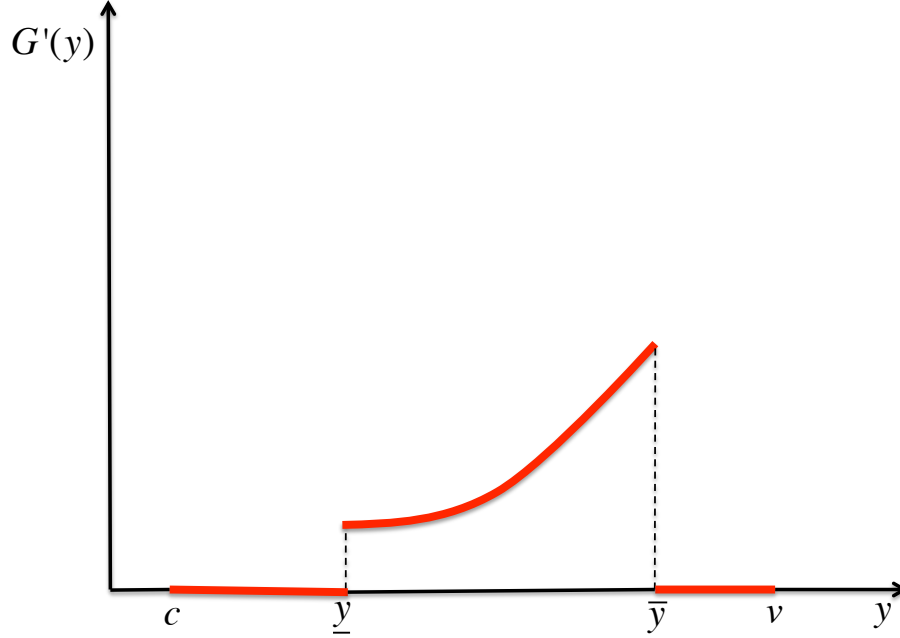
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Figure 2: Small seller's price distribution

price posted by the large seller for $x \in [\underline{x}, \overline{x})$ and it is equal to the price posted by the large seller for $x = \overline{x} = x^*$. For $x \in [\underline{x}, \overline{x}]$, the reservation price is strictly increasing in x, but less than one for one. Moreover, using (20), one can show that the reservation price is strictly concave in x. The reservation price is informative about the buyer's expected surplus since $E_x[V(x)] = u - E_x[R(x)]$. Since $R(x)$ is strictly increasing in x and x takes values between $\underline{x}$ and $\overline{x}$, it follows that buyer's expected surplus is bounded below by $u - R(\overline{x})$ and bounded above by $u - R(\underline{x})$. Since $R(\overline{x}) = \overline{x} \in (c, u)$ and $R(\underline{x}) \in (c, u)$, it follows that the buyer's expected surplus is strictly greater than 0 and strictly smaller than $v - c$. That is, the equilibrium of our market with search frictions is somewhere between a perfectly competitive equilibrium and a pure monopoly equilibrium.

At the beginning of every market day, the large seller draws its price from the distribution F and the small sellers post prices according to the distribution G. In a positive fraction of days, the large seller draws a price x equal to $\overline{x}$. When $x = \overline{x}$, the buyer's reservation price takes its highest value $R(\overline{x})$ and, hence, the buyer's surplus takes its lowest value $u - R(\overline{x})$. Moreover, when $x = \overline{x}$, none of the small sellers is priced out of the market and, hence, every meeting between a buyer and a seller leads to a trade. Since all meetings between buyers and sellers are associated with the same positive gains from
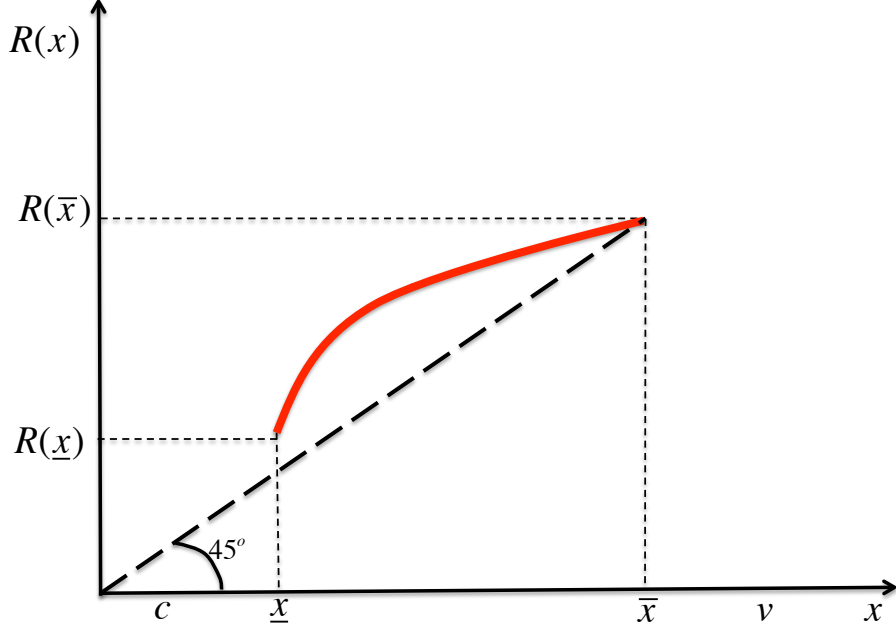
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Figure 3: Buyer's reservation price

trade $u - c > 0$, the market outcome is efficient when $x = \overline{x}$. From time to time, the large seller draws a price x below $\overline{x}$. When $x < \overline{x}$, the buyer's reservation price takes the value $R(x)$ and the buyer's surplus takes the value $u - R(x)$, which is higher the lower is x. Moreover, when $x < \overline{x}$, a fraction of small sellers is priced out of the market and, hence, not all meetings between buyers and sellers lead to trade. Hence, when $x < \overline{x}$, the market outcome is inefficient and the extent of the inefficiency is higher the lower is x.

The behavior of the equilibrium depends critically on the degree of market power of the large seller—as measured by the parameter α—and on the extent of search frictions—as measured by the parameter λ. First, consider the behavior of the equilibrium as the market power of the large seller vanishes, in the sense that α goes to zero. In this case, the price distributions of the large and small sellers are such that

$$\lim_{\alpha \to 0} \overline{x} = u, \quad \lim_{\alpha \to 0} \mu(\overline{x}) = 1, \tag{25}$$

$$\lim_{\alpha \to 0} \overline{y} = u, \quad \lim_{\alpha \to 0} \underline{y} = u. \tag{26}$$

The buyer's expected surplus is such that

$$0 = u - \lim_{\alpha \to 0} R(\overline{x}) \leq \lim_{\alpha \to 0} E_x[V(x)] \leq u - \lim_{\alpha \to 0} R(\underline{x}) = 0. \tag{27}$$

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The limits in (25)-(27) reveal that, as the market power of the large seller vanishes, all the sellers post a price equal to the buyers' valuation and the buyers' expected surplus is equal to zero. That is, as the market power of the large seller vanishes, the equilibrium outcomes converge to those of a pure monopoly. This finding is not surprising because, as $\alpha \to 0$, our environment converges to the one studied by Diamond (1971). Indeed, as $\alpha \to 0$, the large seller has no impact on the buyer's reservation price and, in turn, on the fraction of small sellers that are priced out of the market. For this reason, the large seller has no incentive to post any price below the buyer's reservation price. Moreover, as $\alpha \to 0$, the small sellers face no uncertainty about the realization of the reservation price because this price is unaffected by the large seller. For this reason, the small sellers also have no incentive to post any price below the buyer's reservation price. And, as pointed out by Diamond (1971), when all the sellers post the reservation price, the buyers' reservation price must be equal to u and their surplus must be equal to zero.

Next, consider the behavior of the economy when the market power of the large seller becomes complete, in the sense that α goes to one. In this case, the price distributions of the large and small sellers are such that

$$\lim_{\alpha \to 1} \overline{x} = u, \quad \lim_{\alpha \to 1} \underline{x} = u, \tag{28}$$

$$\lim_{\alpha \to 1} \overline{y} = u, \quad \lim_{\alpha \to 1} \underline{y} = u. \tag{29}$$

The buyer's expected surplus is such that

$$0 = u - \lim_{\alpha \to 1} R(\overline{x}) \leq \lim_{\alpha \to 1} E_x[V(x)] \leq u - \lim_{\alpha \to 1} R(\underline{x}) = 0. \tag{30}$$

The limits in (28)-(30) reveal that, when the market power of the large seller becomes complete, all the sellers post prices equal to the buyers' valuation and the buyers capture none of the gains from trade. That is, when the market power of the large seller becomes complete, the equilibrium outcomes converge to those of a pure monopoly. This finding is intuitive. As $\alpha \to 1$, the large seller has an impact on the buyer's reservation price and on the fraction of small sellers that are priced out of the market. However, the fraction of small sellers that are priced out of the market has no impact on the number of trades that the large seller makes. For this reason, the large seller has no incentive to post any price below the buyers' reservation price. Similarly, the small sellers have no incentive to post any price below the buyers' reservation price because the large seller has no incentive to price them out of the market. Again, as all sellers post the reservation price, the buyers'
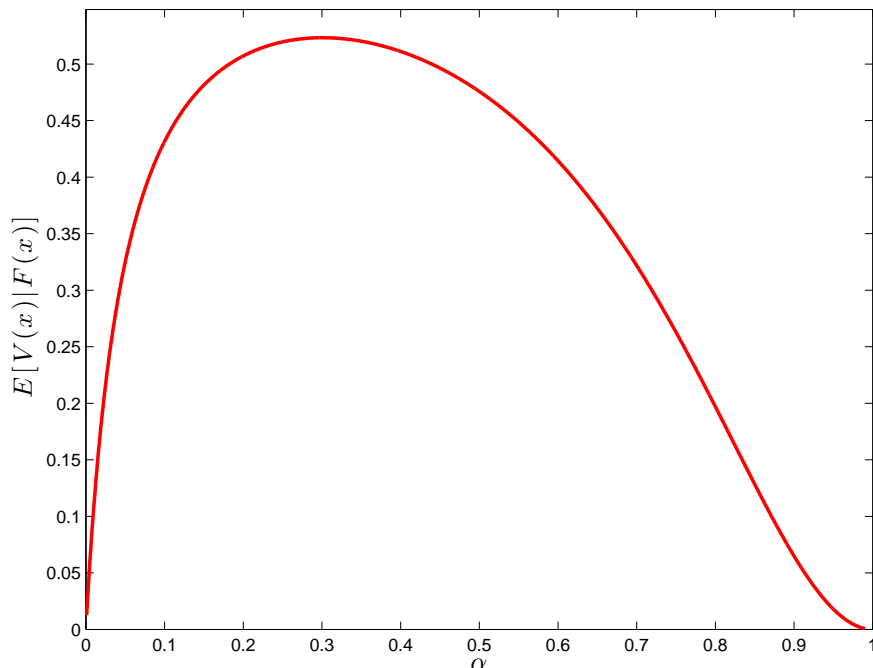
Figure 4: Buyer's surplus and market power

reservation price is u and their surplus is zero.

Overall, we have shown that the buyer's surplus is strictly positive for any $\alpha \in (0, 1)$ and converges to zero for both $\alpha \to 0$ and $\alpha \to 1$. For $\alpha \to 0$, the fraction of the gains from trade accruing to the buyers goes to zero because our environment converges to the one studied by Diamond (1971). For $\alpha \to 1$, the fraction of the gains from trade accruing to the buyers goes to zero because the large seller effectively becomes a monopolist. For $\alpha \in (0, 1)$, the incentives of the large seller to price the small sellers out of the market and the incentive of the small sellers to avoid being priced out of the market by the large seller lead to lower prices and allow buyers to capture some of the gains from trade. Therefore, the fraction of the gains from trade captured by the buyers is nonmonotonic in the market power of the large seller. In this sense, one can view the Diamond Paradox as the limit of a paradoxical nonmonotonic relationship between the extent of competition between sellers and the degree of market power of the large seller. Figure 4 illustrates this nonmonotonic relationship for $u = 1$, $c = 0.2$, $\rho = 0.1$ and $\lambda = 2$.

For any $\alpha \in (0, 1)$, the extent of competition varies in a natural way depending on the extent of search frictions. Consider the behavior of the economy as search frictions

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vanish, in the sense that $\lambda \to \infty$. In this case, the price distributions of the large and small sellers are such that

$$\lim_{\lambda \to \infty} \bar{x} = c, \quad \lim_{\lambda \to \infty} \underline{x} = c, \tag{31}$$

$$\lim_{\lambda \to \infty} \bar{y} = c, \quad \lim_{\lambda \to \infty} \underline{y} = c. \tag{32}$$

The buyer's expected surplus is such that

$$u - c = u - \lim_{\lambda \to \infty} R(\bar{x}) \leq \lim_{\lambda \to \infty} E_x[V(x)] \leq u - \lim_{\lambda \to \infty} R(\underline{x}) = u - c. \tag{33}$$

The limits in (31)-(33) reveal that, as search frictions vanish, all the sellers post prices equal to the marginal cost and the buyers capture all of the gains from trade. That is, as search frictions vanish, the equilibrium outcomes converges to those that would obtain under perfect competition in a frictionless market. Intuitively, as $\lambda \to \infty$, buyers meet sellers faster and faster and, for any nondegenerate distribution of prices, the reservation price gets closer and closer to the lowest posted price. In turn, this induces sellers to push prices towards the marginal cost c. Notice that this argument does not hold when $\alpha = 0$. Indeed, when $\alpha = 0$, the price distribution is degenerate and, hence, the velocity at which buyers meet sellers has no effect on the buyers' reservation price.

Finally, consider the behavior of the economy as search frictions become infinitely large, in the sense that $\lambda \to 0$. In this case, the price distributions of the large and small sellers are such that

$$\lim_{\lambda \to 0} \bar{x} = u, \quad \lim_{\lambda \to 0} \underline{x} = u, \tag{34}$$

$$\lim_{\lambda \to 0} \bar{y} = u, \quad \lim_{\lambda \to 0} \underline{y} = u. \tag{35}$$

The buyer's expected surplus is such that

$$0 = u - \lim_{\lambda \to 0} R(\bar{x}) \leq \lim_{\lambda \to 0} E_x[V(x)] \leq u - \lim_{\lambda \to 0} R(\underline{x}) = 0. \tag{36}$$

The limits in (34)-(36) reveal that, as search frictions become infinitely large, all the sellers post prices equal to the buyers' valuation and the buyers capture none of the gains from trade. That is, as search frictions become infinitely large, the equilibrium outcomes converge to those of a pure monopoly. Intuitively, as search frictions become larger, buyers meet sellers at a lower rate and, for any nondegenerate distribution of prices, their reservation price increases. In turn, this induces the large and the small sellers to push their prices toward the buyer's valuation u.

Figure 5 illustrates the relationship between the fraction of the gains from trade accru-
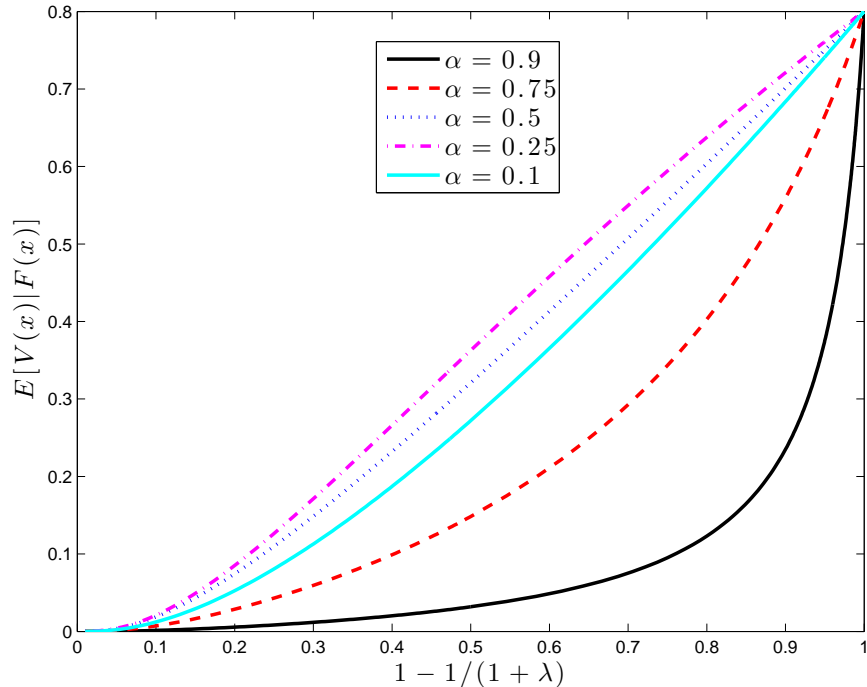
Figure 5: Buyer's surplus and search frictions

ing to the buyers and the extent of search frictions for different values of α, given $u = 1$, $c = 0.2$, and $\rho = 0.1$. For any value of α greater than zero and smaller than one, buyers capture all of the gains from trade when search frictions vanish. Only for α equal to 0 or α equal to 1, buyers do not capture any of the gains from trade irrespective of how small search frictions might be. Diamond (1971) stressed the existence of a discontinuity in equilibrium outcomes between the case in which search frictions are arbitrarily small and the case in which the market is frictionless. Our results show that this discontinuity is not a robust feature of the equilibrium. It only exists when the large seller has either no market power or complete market power.

4 Conclusions

This paper studied equilibrium pricing in a product market with search frictions. As in Diamond (1971), buyers search sequentially for sellers. In contrast to Diamond (1971), buyers do not meet all sellers with the same probability. In particular, a fraction of the buyers' meetings involves one particular large seller, while the remaining fraction involves one out of a continuum of small sellers. We established the existence and uniqueness of the

equilibrium, and we fully characterized its properties. We found that the equilibrium is such that both the large and the small sellers play mixed pricing strategies. The fact that small sellers play mixed strategies implies that there is price dispersion in equilibrium. The fact that the large seller plays mixed strategies implies that there is variation in prices and quantities from one day to the next. Buyers capture a positive fraction of the gains from trade as long as the market power of the large seller—as measured by the parameter α—is positive but incomplete. When this is the case, the fraction of the gains from trade accruing to the buyers depends in a natural way on the extent of search frictions—as measured by the parameter λ. Specifically, buyers capture all of the gains from trade when search frictions vanish, and they capture none of the gains from trade when search frictions become infinitely large. It is only when the large seller has no market power at all (the case considered in Diamond 1971) or when he has complete market power that buyers do not capture any of the gains from trade independently of how small search frictions might be. Therefore, the Diamond Paradox is a non-generic outcome in markets with sequential search.

The only difference between the environment considered in this paper and the one analyzed by Diamond is the presence of a large seller. The large seller's price has an impact on the buyers' decision of whether or not to purchase from the small sellers and, hence, the number of buyers who meet and purchase from the large seller. This mechanism is sufficient to break the Diamond Paradox. In fact, if all sellers posted the monopoly price, the large seller could price them out of the market by lowering his price infinitesimally. The mechanism is also sufficient to generate price dispersion. In fact, because the large seller tries to price the small sellers out of the market and the small sellers try not to be priced out of the market, the sellers are playing a game of cat and mouse whose only equilibrium involves mixed strategies and, hence, price dispersion.

The theory of price dispersion advanced in this paper seems quite general. As the discussion above suggests, our theory of price dispersion follows immediately from the presence of a large seller whose pricing decision impacts the buyers' search strategy and, in turn, the number of buyers who visit his store. Hence, our theory should generalize to an environment in which there are several large seller and a continuous fringe of small sellers, as well as to an environment in which all sellers are large. Similarly, our theory should generalize to an environment in which sellers are ex-ante identical and choose their size through an ex-ante investment. However, one might still wonder whether it is reasonable to assume that there are some sellers whose pricing decisions can affect the

buyers' reservation strategy and, in turn, the fraction of buyers who visit their stores. First, for a seller to impact the buyers' reservation price, it has to be the case that (some) buyers meet that seller with positive probability. This condition seems reasonable. Second, for a seller to impact the buyers' reservation price, it has to be the case that (some) buyers know the price distribution when they embark in their search. This condition seems at least plausible. For example, it may be the case that some buyers learn the distribution of prices on the Internet before leaving home in the morning and, afterwards, they meet a sequence of sellers on their way to and from work. Third, for a seller's price to impact the number of buyers who visit his store, it has to be the case that some other sellers cannot respond to his pricing decision. This condition seems plausible, as any delay in price adjustment would suffice.

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Appendix

A   Proof of Lemma 1

(i) A small seller never posts a price strictly lower than c, as this would give him strictly negative profits. Hence, $G(c-) = 0$. For the same reason, the large seller never posts a price strictly lower than c. Hence, $x \geq c$. Now, fix $x \geq c$. The buyer's value of searching V is such that

$$\rho V = \begin{cases} \Phi(V) + \lambda\alpha(u - x - V) & \text{if } V < u - x, \\ \Phi(V) & \text{if } V \geq u - x, \end{cases} \tag{A1}$$

where

$$\Phi(V) = \int \max\{u - y - V, 0\} dG(y). \tag{A2}$$

Let $LHS(V|x)$ denote the left-hand side of (A1). The function $LHS(V|x)$ is a continuous, strictly increasing function of V such that $LHS(0|x) = 0$ and $LHS(u - c|x) = \rho(u - c) > 0$. Let $RHS(V|x)$ denote the right-hand side of (xx). The function $RHS(V|x)$ is a continuous function of V because $\Phi(V)$ is a continuous function. The function $RHS(V|x)$ is decreasing in V because both $\Phi(V)$ and $\lambda\alpha(u - x - V)$ are decreasing in V. Moreover, $RHS(0|x) \geq 0$ because $\Phi(0) \geq 0$ and and $RHS(u - c|x) = 0$ because $G(c-) = 0$ implies $\Phi(u - c) = 0$.

The properties of $LHS(V|x)$ and $RHS(V|x)$ can be used to characterize the solution to (A1). First, there exists a unique V that solves (A1) since $LHS(V|x)$ is strictly increasing in V and $RHS(V|x)$ is decreasing in V. Let us denote as $V(x)$ the solution to (A1). Second, $V(x) \in [0, u-c)$ since $LHS(0|x) \leq RHS(0|x)$ and $LHS(u-c|x) > RHS(u-c|x)$. Third, $V(x)$ is decreasing in x since $LHS(V|x)$ is independent of x and $RHS(V|x)$ is decreasing in x.

Notice that $V(x) \in [0, u-c)$ implies that $V(x) < (u-x)$ for $x = c$ and $V(x) \geq (u-x)$ for $x = u$. Since $V(x)$ is continuous in x, there exists at least one $x^* \in (c, u]$ such that $V(x^*) = u - x^*$. Let x_1 be equal to one of these x^*s. Then, $LHS(V(x_1)|x_1) = \rho V(x_1)$, $RHS(V(x_1)|x_1) = \Phi(V(x_1))$ and $LHS(V(x_1)|x_1) = RHS(V(x_1)|x_1)$. Now, consider an arbitrary $x_2 > x_1$. Then, $LHS(V(x_1)|x_2) = \rho V(x_1)$ and $RHS(V(x_1)|x_2) = \Phi(V(x_1))$ since $V = u - x_1$ implies $V > u - x_2$. Therefore, $LHS(V(x_1)|x_2) = LHS(V(x_1)|x_1)$ and $RHS(V(x_1)|x_2) = RHS(V(x_1)|x_1)$, which implies $LHS(V(x_1)|x_2) = RHS(V(x_1)|x_2)$ and $V(x_2) = V(x_1)$. Since $x_1 = x^*$ and x_2 was chosen arbitrarily, it follows that $V(x) = V(x^*)$ for all $x \geq x^*$. Since $V(x^*) = (u - x^*)$ and $V(x) = V(x^*)$ for all $x > x^*$, it follows

that $V(x) > u - x$ for all $x > x^*$. Since $V(x^*) = (u - x^*)$ and $V(x) > (u - x)$ for any $x > x^*$, there is a unique x^* such that $V(x^*) = (u - x^*)$.

Now, consider x_1 and x_2 such that $c \leq x_1 < x_2 \leq x^*$. Since $x_1 < x^*$, $V(x) < u - x$ and

$$\rho V(x_1) = \Phi(V(x_1)) + \lambda \alpha (u - x_1 - V(x_1)). \tag{A3}$$

Similarly, since $x_2 < x^*$, $V(x) < u - x$ and

$$\rho V(x_2) = \Phi(V(x_2)) + \lambda \alpha (u - x_2 - V(x_2)). \tag{A4}$$

After subtracting (A3) from (A4) and grouping terms, we obtain

$$V(x_1) - V(x_2) = \frac{\Phi(V(x_1)) - \Phi(V(x_2)) + \lambda \alpha [x_2 - x_1]}{\rho + \lambda \alpha}. \tag{A5}$$

Since $\Phi(V)$ is decreasing in V and $V(x)$ is decreasing in x, $\Phi(V(x_1)) < \Phi(V(x_2))$. Therefore, we have

$$V(x_1) - V(x_2) \leq \frac{\lambda \alpha}{\rho + \lambda \alpha} (x_2 - x_1) < x_2 - x_1. \tag{A6}$$

Since $V(x)$ is decreasing in x, $V(x_1) \geq V(x_2)$. If $V(x_1) = V(x_2)$, then $\Phi(V(x_1)) = \Phi(V(x_2))$ and the right-hand side of (xx) is strictly positive. Hence, we must have

$$0 < V(x_1) - V(x_2). \tag{A7}$$

Parts (i), (ii) and (iii) in Lemma 1 follow immediately from the properties of $V(x)$ and $R(x) = u - V(x)$. ■

B Proof of Lemma 3

(i) We want to establish that $\bar{x} = x^*$ in any equilibrium. Let (R, F, G) denote an equilibrium. First, suppose $\bar{x} > x^*$. The profit of the large seller is given by

$$L(x) = \frac{b \lambda \alpha \mathbf{1}[x \leq R(x)](x - c)}{\rho + \lambda \alpha \mathbf{1}[x \leq R(x)] + \lambda(1 - \alpha)G(R(x))}. \tag{B1}$$

Notice that $L(\bar{x}) = 0$ because $\bar{x} > x^*$ implies $\mathbf{1}[\bar{x} \leq R(\bar{x})] = 0$. In contrast, $L(x^*) > 0$ because $\mathbf{1}[x^* \leq R(x^*)] = 1$ and $x^* > c$. Therefore, $L(\bar{x}) < L(x^*) \leq L^*$. However, since (R, F, G) is an equilibrium, $L(\bar{x}) = L^*$. Thus, we have reached a contradiction and $\bar{x}$

must be smaller or equal to x^*.

Next, suppose $\bar{x} < x^*$. The profit of the small seller is given by

$$S(y) = \int \frac{b\lambda(1-\alpha)\mathbf{1}[y \le R(x)](y-c)}{\rho + \lambda\alpha\mathbf{1}[x \le R(x)] + \lambda(1-\alpha)G(R(x))} dF(x). \tag{B2}$$

Notice that $S(y) = 0$ for all $y > R^{-1}(\bar{x})$. In contrast, $S(R(c)) > 0$ because $F(c-) = 0$ and $R(c) > c$. Therefore, $S(y) < S(R(c)) \le S^*$ for all $y > R^{-1}(\bar{x})$. Since $S(y) = S^*$ for all $y \in \mathcal{G}$, it follows that $G(R^{-1}(\bar{x})) = 1$.

For all $x \in [\bar{x}, x^*]$, $G(R(x)) = 1$ and, hence, the profit of the large seller is such that

$$L(x) = \frac{b\lambda\alpha(x-c)}{\rho + \lambda}. \tag{B3}$$

The above expression implies that $L(\bar{x}) < L(x^*) \le L^*$. However, since (R, F, G) is an equilibrium, $L(\bar{x}) = L^*$. Thus, we have reached a contradiction and $\bar{x}$ must be greater or equal to x^*.

(ii) We want to establish that $\bar{y} = R(\bar{x})$ in any equilibrium. Let (R, F, G) denote an equilibrium. We have already shown above that $\bar{y} > R(\bar{x})$ is not consistent with equilibrium. Now, suppose $\bar{y} < R(\bar{x})$. For all $x \in [R^{-1}(\bar{y}), \bar{x}]$, $G(R(x)) = 1$ and $\mathbf{1}[x \le R(x)] = 1$. Hence, for all $x \in [R^{-1}(\bar{y}), \bar{x}]$, the profit of the large seller is given by

$$L(x) = \frac{b\lambda\alpha}{\rho + \lambda}(x - c). \tag{B4}$$

The above expression implies that $L(R^{-1}(\bar{y})) < L(\bar{x}) \le L^*$. Since $L(x) = L^*$ for all $x \in \mathcal{F}$, $L(R^{-1}(\bar{y})) < L^*$ implies that $F(R^{-1}(\bar{y})-) = F(\bar{x}-)$.

Since $F(R^{-1}(\bar{y})-) = F(\bar{x}-)$, the profit of the small seller is such that

$$\begin{aligned} S(\bar{y}) &= \frac{b\lambda(1-\alpha)}{\rho + \lambda}(\bar{y} - c)\mu(\bar{x}) \\ &\le \frac{b\lambda(1-\alpha)}{\rho + \lambda}(R(\bar{x}) - c)\mu(\bar{x}) = S(R(\bar{x})), \end{aligned} \tag{B5}$$

where $\mu(\bar{x})$ denotes the probability that the large seller posts the price $\bar{x}$. If $\mu(\bar{x}) > 0$, $S(\bar{y}) < S(R(\bar{x})) \le S^*$. If $\mu(\bar{x}) = 0$, $S(\bar{y}) = S(R(\bar{x})) = 0 < S^*$. However, since (R, F, G) is an equilibrium, $S(\bar{y}) = S^*$. Thus, we have reached a contradiction and $\bar{y}$ must be greater or equal to $R(\bar{x})$.

(iii) We want to establish that $\underline{y} = R(\underline{x})$ in any equilibrium. Let (R, F, G) denote an equilibrium. First, suppose $\underline{y} < R(\underline{x})$. For all $y \in [\underline{y}, R(\underline{x})]$, $F(R^{-1}(y)-) = 0$ and, hence,

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the profit of the small seller is given by

$$S(y) = \frac{b\lambda(1-\alpha)}{\rho + \lambda}(y - c). \tag{B6}$$

The above expression implies that $S(\underline{y}) < S(R(\underline{x})) \leq S^*$. However, since (R, F, G) is an equilibrium, $S(\underline{y}) = S^*$. Thus we have reached a contradiction and $\underline{y}$ must be greater or equal to $R(\underline{x})$.

Next, suppose $\underline{y} > R(\underline{x})$. For all $x \in [\underline{x}, R^{-1}(\underline{y}))$, $G(R(x)) = 0$ and, hence, the profit of the large seller is given by

$$L(x) = \frac{b\lambda\alpha}{\rho + \lambda\alpha}(x - c). \tag{B7}$$

The above expression implies that $L(\underline{x}) < L(R^{-1}(\underline{y}) - \epsilon)$ for any $\epsilon \in (0, R^{-1}(\underline{y}) - \underline{x})$. However, since (R, F, G) is an equilibrium, $L(\underline{x}) = L^* \geq L(R^{-1}(\underline{y}) - \epsilon)$. Thus we have reached a contradiction and $\underline{y}$ must be smaller or equal to $R(\underline{x})$. ∎

C Proof of Lemma 4

(i) We want to establish that the distribution G has no gaps in equilibrium. Let (R, F, G) denote an equilibrium. On the way to a contradiction, suppose that there is a gap in the distribution G between the prices y_1 and y_2, with $y_1 \in \mathcal{G}$, $y_2 \in \mathcal{G}$ and $y_1 < y_2$. For all $x \in [R^{-1}(y_1), R^{-1}(y_2))$, $G(R(x)) = G(y_1)$ and $\mathbf{1}[x \leq R(x)] = 1$. Therefore, for all $x \in [R^{-1}(y_1), R^{-1}(y_2))$ the profit of the large seller is given by

$$L(x) = \frac{b\lambda\alpha(x - c)}{\rho + \lambda\alpha + \lambda(1 - \alpha)G(y_1)}. \tag{C1}$$

The above expression implies that $L(x)$ is strictly increasing over the interval $[R^{-1}(y_1), R^{-1}(y_2))$. In turn, this implies that $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$.

Since $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$, for all $y \in [y_1, y_2]$ the profit of the small seller is given by

$$S(y) = \int_{R^{-1}(y_1)}^{\bar{x}} \frac{b\lambda(1 - \alpha)(y - c)}{\rho + \lambda\alpha + \lambda(1 - \alpha)G(R(x))} dF(x). \tag{C2}$$

If $F(R^{-1}(y_1)-) < 1$, $S(y_1) < S(y_2) \leq S^*$. If $F(R^{-1}(y_1)-) = 0$, $S(y_1) = S(y_2) = 0 < S^*$. However, since (R, F, G) is an equilibrium, $S(y_1) = S^*$. Thus we have reached a contradiction and there cannot be a gap in the distribution G.

(ii) We want to establish that the distribution G has no mass points in equilibrium. Let (R, F, G) denote an equilibrium. On the way to a contradiction, suppose that there is a

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mass point in the distribution G at the price y_1, with $y_1 < \bar{y}$. In this case, the profit of the large seller has a discontinuity at $R^{-1}(y_1)$. In fact, since $G(y_1-) < G(y_1)$, there is an $\epsilon > 0$ such that

$$
\begin{aligned}
L(R^{-1}(y_1) - \epsilon) &= \frac{b\lambda\alpha(R^{-1}(y_1) - \epsilon - c)}{\rho + \lambda\alpha + \lambda(1-\alpha)G(y_1 - \epsilon)} \\
&> \frac{b\lambda\alpha(R^{-1}(y_1) - c)}{\rho + \lambda\alpha + \lambda(1-\alpha)G(y_1)} = L(R^{-1}(y_1)).
\end{aligned}
\tag{C3}
$$

The discontinuity in the profit of the large seller implies that $L(x) < L(R^{-1}(y_1) - \epsilon)$ for all $x \in [R^{-1}(y_1), R^{-1}(y_2)]$ for some $y_2 \in (y_1, \bar{y})$. In turn, this implies that $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$.

Since $F(R^{-1}(y_1)-) = F(R^{-1}(y_2)-)$, for all $y \in [y_1, y_2]$ the profit of the small seller is given by

$$
S(y) = \int_{R^{-1}(y_1)}^{\bar{x}} \frac{b\lambda(1-\alpha)(y-c)}{\rho + \lambda\alpha + \lambda(1-\alpha)G(R(x))} dF(x).
\tag{C4}
$$

If $F(R^{-1}(y_1)-) < 1$, $S(y_1) < S(y_2) \le S^*$. If $F(R^{-1}(y_1)-) = 0$, $S(y_1) = S(y_2) = 0 < S^*$. However, since (R, F, G) is an equilibrium, $S(y_1) = S^*$. Thus we have reached a contradiction and there cannot be a mass point at $y_1 < y^*$. A similar argument can be used to show that there cannot be a mass point at $y_1 = y^*$. ∎

D   Proof of Lemma 5

(i) We want to establish that the distribution F has no gaps in equilibrium. Let (R, F, G) denote an equilibrium. On the way to a contradiction, suppose that there is a gap in the distribution F between the prices x_1 and x_2, with $x_1 \in \mathcal{F}$, $x_2 \in \mathcal{F}$ and $x_1 < x_2$. For all $y \in [R(x_1), R(x_2))$, $F(x_1) = F(x_2-)$. Therefore, for all $y \in [R(x_1), R(x_2)]$, the profit of the small seller is given by

$$
S(y) = \int_{x_1}^{\bar{x}} \frac{b\lambda(1-\alpha)(y-c)}{\rho + \lambda\alpha + \lambda(1-\alpha)G(R(x))} dF(x).
\tag{D1}
$$

The above expression implies that $S(y)$ is strictly increasing over the interval $[R(x_1), R(x_2)]$. In turn, this implies that $G(R(x_1)-) = G(R(x_2)-)$. That is, the distribution G has a gap between the prices $R(x_1)$ and $R(x_2)$ Since, this contradicts Lemma 4, the distribution F cannot have a gap.

(ii) We want to establish that the distribution F has a mass point only at $\bar{x}$. Let (R, F, G)

denote an equilibrium. To establish that F has a mass point at $\bar{x}$, suppose it does not. Then, the probability that the large seller posts a price strictly smaller than $\bar{x}$ is one, i.e. $F(\bar{x}-) = 1$. In turn, this implies that the profit of a small seller posting the price $\bar{y} = R(\bar{x})$ is zero, i.e. and $S(\bar{y}) = 0$. However, since (R, F, G) is an equilibrium and $\bar{y} \in \mathcal{G}$, $S(\bar{y}) = S^* > 0$. Thus, we have reached a contradiction and F must have a mass point at $\bar{x}$.

To establish that F does not have any other mass points, suppose there is a mass point at some price $x_1 < \bar{x}$. Then, the profit of the small seller is discontinuous at $R(x_1)$. In particular, $S(R(x_1)) > S(R(x_1)+)$. Hence, there exists an $R(x_2) \in (R(x_1), \bar{y})$ such that $S(y) < S(R(x_1)) \le S^*$ for all $y \in (R(x_1), R(x_2))$. Since $S(y) = S^*$ for all $y \in \mathcal{G}$, the previous inequality implies $G(R(x_1)) = G(R(x_2))$ That is, the distribution G has a gap between the prices $R(x_1)$ and $R(x_2)$. Since, this contradicts Lemma 4, the distribution F cannot have a mass point at $x < \bar{x}$. ∎