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Modeling Labor Markets in Macroeconomics: Search and Matching*

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Abstract

We present and discuss the simple search and matching model of the labor market against the background of developments in modern macroeconomics. We derive a simple representation of the model in a general equilibrium context and how the model can be used to analyze various policy issues in labor markets and monetary policy.

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1 Introduction

Macroeconomic models of the labor market serve two purposes. First, they aim to explain both the cyclical and the long-run, or secular, movements of key labor market variables, such as the unemployment rate, hours worked, or job vacancies. Understanding the behavior of these variables is crucial for assessing the welfare implications of labor market policies and institutions and the effectiveness of macroeconomic stabilization policies. Second, the labor market serves as a key mechanism for the transmission of monetary and fiscal policy and provides information about the state of the economy upon which policymakers base their decisions.

This chapter discusses the currently prevailing approach for thinking about the macroeconomics of labor markets: the search and matching model. This model describes the dynamics of employment and unemployment as the outcome of worker flows, which result from job creation and separation decisions of firms and workers. As a quantitative theory building on explicit microfoundations, the model is designed to account for empirical labor market facts and to be suitable for counterfactual and normative policy analysis. In contrast, the standard neoclassical model of perfectly competitive labor markets is ill-suited for some of these tasks since it does not allow for the existence of involuntary unemployment easily.[1] Moreover, it does not offer predictions for a host of labor market variables that are of practical importance for policymakers and researchers alike.

The search and matching model is built on the notion of matching frictions that are present in real world markets. These arise from heterogeneity and imperfect information about potential trading partners. Examples for such heterogeneity are the skills of workers and the characteristics of products. In the context of labor markets, such frictions are the central departure from the competitive paradigm, since they force workers and firms to engage in a two-sided search process. Once a match has formed, there are rents to be shared between the trading partners, as searching for another partner is time consuming and costly, and therefore worth avoiding. To determine how rents are shared, the bilateral monopoly situation between a worker and a firm needs to be resolved by some assumption on how bargaining takes place. Thus competitive forces do not suffice to determine wages.

[1] A standard approach to the issue of unemployment in neoclassical frameworks (Hansen, 1985) is that workers are chosen by a lottery to participate in employment whereby the success probability is a function of the equilibrium wage.

The resulting labor market model that includes search and matching frictions typically consists of two equations at its core. One equilibrium relationship describes the dynamics of employment as a function of the job matching and separation process that governs the flows into and out of unemployment. The other describes the hiring choice given the anticipated wage and prevailing as well as expected labor and product market conditions. The two equations provide a rich set of empirical implications concerning the short- and long-run dynamics of the labor market. The long-run, steady-state equilibrium features a notion of the natural rate of unemployment and of the associated worker flows and job finding rates, which are determined by a variety of structural factors. The short-run response to aggregate shocks exhibits vacancy and unemployment fluctuations that can be compared to the data. Therefore, the guiding principle of our article is how the behavior of key aggregate labor market variables can be understood using tools and insights from both economic theory and empirical analysis.

What emerges from the interplay of the actors and their choices is unemployment as an equilibrium outcome in the sense that it is compatible with the actors' choices and that there is no incentive to deviate from the chosen policies. The key insight is that unemployment is not something that just happens to the laborforce, but that it is the outcome of the decision process of rational actors, firms, and households, which has to be understood as such for the construction and evaluation of labor market policies. This framework entertains no notion of disequilibrium in that a short end of the market rations quantity. On the contrary, what generates measured unemployment, defined as the state of a worker who does not generate wage income from an employment relationship, is the presence of real rigidities in the form of costs and inefficiencies in the process of bringing workers and firms together.

An important aspect of many current approaches to analyze the macroeconomics of labor markets is that the search and matching model is treated as a data-generating process. This means that the model provides a time-series representation for the labor market variables of interest, which can then be compared to the relevant data at various levels of statistical complexity. This can range from comparing the model's predictions for key statistics, such as the variance of the unemployment rate, to its observed counterpart in the data, to a full-blown structural estimation of the labor market model. The model thus imposes theoretical structure on the data, without which we can at best derive correlation, but will not be able to ascertain causation.

The latter is, of course, the crucial element in analyzing economic policies. Although our article largely proceeds along a theoretical perspective, it should be understood that there are always empirical considerations in the background against which the success of a model should be evaluated.

An additional attractive feature of the search and matching approach is its modular nature. The two-equation system, which can be described as 'labor supply' and 'labor demand' is virtually self-contained, in that it can be solved (almost) independently from the rest of the economy. In turn, the search and matching model can be appended to frameworks that incorporate other crucial aspects of economic decisions, for instance, the New Keynesian model of monopolistic competition under sticky prices. At the end of the chapter, we demonstrate how to synthesize these two models, thus providing a glimpse of a current frontier of macroeconomic research.

This chapter first gives a brief overview of the development of models of the aggregate labor market in the past, such as the classical model and the Keynesian approach, thereby motivating the need for the development of a search-based approach to labor market analysis. We then introduce a number of stylized facts relevant for labor market analysis and set up the standard search and matching model in its simplest form to derive the key relationships describing labor market equilibrium.[2] Next, we confront the main implications of the model with the data in order to assess its empirical performance. Finally, we close this chapter by introducing the key elements of the basic New Keynesian model, show how to embed in it the search and matching model, and discuss some implications for policy.

2 A Brief History of Thought on the Macroeconomics of Labor Markets

Until the 1970s, there was a dichotomy between the microeconomic and the macroeconomic analysis of the labor market. This made the debate between advocates of different policy views cumbersome. One side, the classical or new classical view, tended to rely on microeconomic arguments and to focus on the equilibrating forces of competitive markets. The other side, with a more traditional Keynesian bent, used

[2] A special feature of our exposition is that we provide it in discrete time. Most surveys and research papers cast the search and matching model in continuous time. Since most of business cycle and also monetary policy analysis is conducted in discrete time models, the labor market has to be described analogously to compare the model's implications with macroeconomic time series.

macroeconomic arguments that were based on empirically observed relationships and strong beliefs about imperfections at work in the labor market. We discuss these different perspectives and then turn to the advances that eventually helped reconcile them. These developments ultimately led to the search and matching model as a microeconomically based theory of the aggregate labor market that can be merged with equally microfounded models of aggregate demand and inflation. We first discuss the competitive model of the labor market, then describe the Keynesian approach to inflation and unemployment (the Phillips curve), discuss the ideas of New Classical economics of the 1970s and the NAIRU, and finally move on to the point where New Keynesian economics and the search and matching approach were developed.[3]

2.1 The Neo-Classical Model

The standard neoclassical model of a competitive labor market assumes that labor markets always clear. The model can therefore not easily rationalize the existence of involuntary unemployment, because wages always equal the marginal product of labor and the marginal rate of substitution, that is, the disutility of working. Ad-hoc wage rigidity may generate unemployment through rationing mechanisms, but this leaves open the question as to why wages are chosen to be sticky, if mutually beneficial choices of firms and workers could eliminate this inefficiency. In terms of matching data, the neoclassical model also has no role for gross flows of workers between employment and nonemployment, an established stylized fact (e.g., Davis, Haltiwanger, and Schuh, 1996), that gives roles to both the hiring and firing margins in order to explain changes in unemployment.

In the simple competitive model, labor supply is derived from household preferences over how much leisure to enjoy and how to consume which has to be paid for by earnings from employment. Labor demand is derived from firms' profit maximization given technology. This results in a relationship where optimal labor input is governed by the equality of the real wage w_t with the marginal product of labor mpl_t; that is, the additional return a worker brings to the firm in terms of additional revenue and the marginal disutility of work mul_t; that is, the loss of utility from leisure that a worker would suffer by supplying an additional unit of labor input:

$$w_t = mpl_t = mul_t. \tag{1}$$

[3] The following two sections draw on the discussion in the monograph by Cahuc and Zylberberg (2000).

This reasoning leads to the insight that labor market outcomes are determined by technology and preferences. In a sense, this is a 'supply-side' view since outcomes are ultimately determined by production possibilities, that is, firm productivity and technology. Moreover, absent any nominal rigidities, real equilibrium is determined solely by supply factors. Changes in the money supply can only have an effect on nominal prices. Thus, money is neutral and the classical dichotomy holds.[4]

What is behind the equation is the idea that for a given wage, workers supply a certain amount of labor. On the other hand, firms demand a certain amount of labor at a prevailing wage. If supply and demand are not equal, the equilibrating mechanism is that the wage will either fall or rise - until equality is restored. What underpins this mechanism is that hiring, firing, and labor adjustment is costless. For instance, if supply exceeded demand, a profit-maximizing firm could lower its wage and still be able to find workers willing to work. Conversely, if demand exceeds supply, firms that have not found enough workers (with the marginal product being higher than the wage) would offer a wage slightly higher than the current one and be able to find additional workers that enter the workforce. Other firms would have to follow suit as otherwise all workers would migrate to the firms offering higher wages. The key to this mechanism is the fungibility of individual workers and the free adjustment

[4] The analytical derivation is straightforward. We assume that that households maximize the combination of (logarithmic) consumption utility and labor disutility:

$$U_t = \log C_t - \frac{\chi}{1+\eta} {N_t}^{1+\eta}$$

subject to a budget constraint:

$$C_t = w_t N_t,$$

where w_t is the real wage. The labor supply condition derives from the utility maximum attained at:

$$w_t = \chi N_t^{\eta}$$

Competitive firms maximize per-period real profits:

$$\Pi_t = p_t Y_t - w_t N_t$$

where p_t is the relative (real) price of the product subject to the production function.

$$y_t = A_t N_t^{1-\alpha}$$

so that optimal labor input is given by the condition

$$w_t = p_t A_t (1-\alpha) N_t^{-\alpha}.$$

of production and labor input, which is impeded in the model with matching frictions.

2.2 Keynesian Ideas: The Phillips Curve and the NAIRU

Macroeconomists working in the Keynesian tradition approached the determination of aggregate labor market outcomes from a different angle. A key assumption in this line of thinking is that prices and wages are sticky, in which case the possibility of both unemployment and monetary non-neutrality arises. Under sluggish adjustment of prices and wages, the so-called Phillips curve can then be used to capture these economic relationships in a straightforward manner. The Phillips curve rests on an observed empirical regularity, namely the comovement of nominal wage growth Δw_t with unemployment u_t. Keynesian macroeconomists loosely connected the change in the nominal wage to current and lagged inflation p_t, the unemployment rate, and the change in productivity Δa_t by means of a simple equation:

$$\Delta w_t = \lambda_0 + (1 - \lambda_1)\Delta p_t + \lambda_1 \Delta p_{t-1} - \lambda_2 u_t + \lambda_3 \Delta a_t, \tag{2}$$

where lower case variables denote logarithms. λ_1 is a measure of nominal rigidity in wages. The higher λ_1 is, the longer wages take to adjust to changes in prices. Rewriting the equation in terms of real wages, yields a notion of real rigidity:

$$\Delta(w_t - p_t) = \lambda_0 - \lambda_1(\Delta p_t - \Delta p_{t-1}) - \lambda_2 u_t + \lambda_3 \Delta a_t. \tag{3}$$

Since the influence of the unemployment rate rises with λ_2, real rigidity can be represented as $1/\lambda_2$,while λ_3 determines how strongly productivity changes translate into wages.

Utilizing the price setting equation above in a relationship between log changes in wages, prices, and productivity, $\Delta w_t - \Delta p_t = \Delta a_t$, we can write:

$$\lambda_1(\Delta p_t - \Delta p_{t-1}) = \lambda_0 - \lambda_2 u_t - (1 - \lambda_3)\Delta a_t \tag{4}$$

Assuming that inflation is constant, i.e., nonaccelerating, we can solve for the NAIRU, the *n*on*a*ccelerating *i*nflation *r*ate of *u*nemployment:

$$\overline{u}_t = \frac{\lambda_0 - (1 - \lambda_3)\Delta a_t}{\lambda_2}, \tag{5}$$

which depends on technology and measures of the various nominal and real rigidities only. Inserting this expression into the equation for the price change results in a convenient representation of the Phillips curve:

$$u_t = \overline{u}_t - \frac{\lambda_1}{\lambda_2}(\Delta p_t - \Delta p_{t-1}). \tag{6}$$

Fluctuations of the unemployment rate around the long-run rate depend on changes in the inflation rate. The more inflation fluctuates, the more does unemployment. If inflation is constant, unemployment is at its natural rate.

This model of the Phillips curve can give a rich account of short- and long-run adjustment in the labor market. Note that in the incarnation presented here, inflation as such does not necessarily reduce unemployment. This reflects the insight that when inflation is stable, wage negotiations incorporate expected price changes. These neutralize the effect that inflation would have on labor costs if wages were fixed. Thus, the model captures the long-run neutrality of money, but still allows for changes in inflation to affect unemployment. In fact, it seemed to suggest to policymakers at that time and exposed to the Keynesian tradition that a central bank could obtain at least in the short-run gains in employment by raising inflation. What is problematic about this approach is that it is unclear how this relationship is determined from first principles, and more specifically, where the parameters come from. Moreover, this NAIRU formulation leaves no room for a determination of expectations. It is these issues that underpinned the rise of New Classical Macroeconomics.

2.3 New Classical Macroeconomics

While the NAIRU constituted the consensus of the 1960s and well into the 1970s among most economists, it left many unsatisfied. On the theoretical side, many doubted that even acceleration of inflation would lead to lower unemployment, for the reason that if economic actors understood the motives of policymakers, they would also anticipate their actions. In this case, preemptive increases of wages would offset potential surprise increases in inflation. To New Classical economists, monetary policy is super-neutral in that even acceleration of inflation has no effect on unemployment. It is also logically impossible, as stipulated by the Lucas-critique, to assess the role of structural policies with the help of reduced-form models. On empirical grounds, the stagflation of 1970s posed another problem for the NAIRU. During that period, U.S. and worldwide inflation was rising at the same time as unemployment was rising. The coincidence of inflation and stagnation - stagflation in the parlance of the time - could not be explained by the model.

There were attempts to reconcile neoclassical market clearing models with Keynesian ideas by imposing wage and price stickiness in essentially static general equilibrium models. The assumption of price rigidity led to one side of the market being rationed, which in turn affected demand and supplies in other markets. Such 'disequilibrium models' (Barro and Grossmann, 1976; Bénassy, 1982) fell in disrepute however, since they fundamentally raised the question as to where the assumed rigidities came from, or why they were not overcome by economic actors if they had large costs in terms of unemployment and output. Furthermore, the models did not allow for expectations to be formed in a manner that was consistent with the actions by the economic agents within the model.

The New Classical approach to macroeconomic fluctuations returned to the basic microeconomic principles of the classical model but introduced the notion of imperfect information to obtain monetary non-neutrality. Agents are assumed to observe aggregate conditions imperfectly; they would therefore set their prices and wages based on their expectations of these conditions. Even if expectations are set rationally, that is, even when agents make no systematic mistakes, monetary shocks can have real effects if they are not anticipated. In contrast, anticipated policies have no effects. Short-run fluctuations are therefore possible, and monetary policy is non-neutral. While this model was a conceptual improvement, especially in terms of clarifying the distinction between anticipated and unanticipated policies, it interpreted most fluctuations as efficient.

Assuming market clearing and rational expectations, New Classical macroeconomics evolved in the 1980s further into what became known as Real Business Cycle theory, using microfounded dynamic stochastic general equilibrium models to explain most fluctuations as the result of shocks to preferences and technologies. Also during the 1980s, Keynesian economists, working on microfounded macroeconomic models, rationalized why profit-maximizing firms and utility-maximizing workers would not find it optimal to adjust prices and wages even if this entailed large macroeconomic costs. In a concerted effort with empirical work that highlighted the presence of rigidities, theoretical researchers introduced models with fixed costs of price adjustment (the so-called menu costs), near-rationality, or price and wage contracts that could not easily be changed in a state-dependent manner. The emerging consensus was that it is sticky prices, rather than imperfect information that gives rise to monetary non-neutralities. This line of research eventually coalesced into New Keynesian economics.

However, the labor market in these models was mainly modeled as competitive, so that unemployment remained largely outside the model.

2.4 The Development of the Search and Matching Approach to Labor Markets

The research agenda that led to the search and matching model was motivated by the goal of developing a theory where unemployment would be an equilibrium outcome. Many of the relevant aspects of labor markets that can determine equilibrium unemployment were already postulated by Friedman (1968) in his definition of the natural rate of unemployment:

> "The natural rate of unemployment is the level that would be ground out by the Walrasian system of general equilibrium equations, provided that there is embedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demand and supplies, the cost of gathering information about job vacancies and labor availabilities, the cost of mobility, and so on."

The focus on market imperfections and the cost of information is central to the development of search and matching models. Thus, the research that followed honed in on the characteristics of search and the market frictions that economic actors face. The one-sided search models by McCall (1970) and Mortensen (1970) can be regarded as first steps in this direction.

The model by McCall (1970) casts a worker's search problem as an optimal stopping problem. Wage offers arrive randomly over time, and workers face the decision of accepting a wage offer or continuing to search. A key insight of this approach is that workers have a reservation wage that serves as a stopping rule for employment search: Workers terminate search for alternative offers if the available wage is above the reservation wage. The interplay of the speed at which job or wage offers arrive and the distribution of wage offers are the key determinants of the duration of unemployment. This model introduces the notion that unemployment may be a productive activity and not necessarily a state of idleness. Yet, this is essentially a partial equilibrium model of labor supply, as the demand side of labor is merely represented by a stochastic process for the arrival of wage offers.

In a next step, Diamond, Mortensen, and Pissarides embedded worker search into a general equilibrium setting where both labor supply and demand interact in a number of individual and joint contributions. Two key innovations are the consideration of two-sided search and the introduction of a matching function that determines the flow of new worker-firm matches per period as a function of the stocks of searching workers and searching firms. The matching function represents the difficulty of searchers to locate a suitable trading partner. This difficulty arises from imperfect information about the characteristics of workers and firms, which consequently leads to a time-consuming search process.

The matching function also incorporates two different types of externalities that are present in matching markets. On the one hand, the more searchers of a specific type (e.g., workers) search for a trading partner of the other type (e.g., firms), the less likely it becomes for the first type to find a partner. This congestion externality is not taken into account by workers or firms. Conversely, a thick-market externality works such that a higher search activity by one type of agent increases the probability of the other type finding a trading partner. Thus the equilibrium in markets with frictions is generally not efficient, and policy interventions in such markets with matching frictions can be meaningful. We describe the specifics of the model in the next section.[5]

3 A Simple Model of the Aggregate Labor Market

We now develop the standard search and matching model along the lines of Pissarides (2000) and use it to illustrate the main conceptual issues that arise in this class of models.

An Intuitive Description The search and matching approach to the labor market treats employment outcomes as the result of decisions made by workers and employers ('firms'). Workers search for employment, employers search for workers. A key aspect is that the decisions of workers and firms are binary in nature: Their choices are between one of two alternatives. Consider a worker who is not in an employ-

[5]While the individual elements of the model are hard to trace back to the three contributors to this line of research, one can argue that the matching friction have first been analyzed by Diamond, the search behavior by Mortensen, and the first steps toward a general equilibrium model with search and matching by Pissarides.

ment relationship. We label this worker for the purposes of this simple exposition as unemployed. He faces a choice between searching for a job and remaining unemployed. While unemployed, a worker may receive unemployment benefits, enjoy leisure or pursue nonmarket activities, and engage in home production. Searching for a job comes with uncertainty, because the outcome of the search process is stochastic. Each worker faces competition from other job seekers, not many open positions may be available, or the skills and experience a specific worker possesses may not be currently in demand. In the simplified framework we discuss below, the outcome of the unemployed worker's decision problem is to always search for a job since it offers the chance of a higher income, whereas the fallback position of remaining unemployed is always available.

Employers face a similar discrete decision problem. In order to satisfy demand and produce output they need to employ workers or, alternatively, they have to hire them. Hiring workers is costly: It involves posting announcements for open positions, evaluating applications, interviewing candidates, and so on. At the same time, it is also uncertain: There may not be enough qualified applicants, there may be competition from other firms for qualified workers, and there may be frictions in the random matching process that disturbs the mutual assignment of employment relationships. If the labor market is tight, that is, if there are relatively many vacancies and relatively few job seekers, then the probability that a firm will hire someone is low. The vagaries of generating new employment relationships by matching job seekers with potential employers is captured by the matching function.

Since hiring is costly, a firm will only undertake recruiting efforts if it expects a successful match to recoup its expenses through furture earned profits. A firm also takes into account that if it hires someone today, it may not have to face tighter hiring conditions tomorrow, which is an implicit cost-saving. Current and future profits are determined by the expected paths of the revenue earned on products sold, net of the wage payments made to workers.

How wages evolve results from the specifics of the bargaining process between workers and firms. Because of the costs of finding a trading partner, matched firms and workers find themselves in a bilateral monopoly situation: Both sides benefit from not having to search for a new partner, thus saving renewed cost of search and of the lost opportunity of producing. How these rents from a match are shared must be specified by the modeler. The typical assumption in this literature is that of a bargaining protocol, such as Nash bargaining, which results in a wage path that shares the present values of staying in a match.

The above description of the mechanics of the search and matching framework can be captured analytically by three equations: the employment accumulation equation, the job creation condition, and a wage equation. We discuss these three equations that jointly determine equilibrium in the labor market now in turn.

A Benchmark Model The dynamics of unemployment can be expressed by a stock-flow identity that relates the stock of employed workers n to the flow of new hires, or matches m:

$$n_t = (1-\rho)n_{t-1} + m_{t-1}. \tag{7}$$

Jobs are destroyed at the separation rate ρ, which captures outflows from employment. The number of new hires is determined as the outcome of the matching process that combines the aggregate stock of vacancies v with the unemployed workers u who search for jobs. The matching function is of the Cobb-Douglas type

$$m_t = \overline{m} v_t^{1-\mu} u_t^{\mu}, \tag{8}$$

with match elasticity $0 < \mu < 1$. The parameter $\overline{m}$ captures the efficiency of the matching process. The total workforce is normalized to one, and the measure of the unemployed is defined as $u_t = 1 - n_t$. The time-consuming nature of the matching process is reflected here by the assumption that matches formed in period $t-1$ become productive in period t.

The employment accumulation equation is in principle an accounting identity. What turns it into something with economic content is the postulate of a matching function. It captures the outcome of the search process as the result of the search inputs decided on by firms and workers. The actual matching function can be regarded as a black box (in the terminology of Petrongolo and Pissarides, 2001) since it does not describe the actual process of matching. Rather, it takes a production function approach that subsumes both the finding and selection aspects of the hiring activity of firms. Nevertheless, the matching function has proved to be remarkably successful in capturing salient labor market behavior.

We can define two additional labor market variables, the hiring and the job-finding rates, that have empirical counterparts. The hiring rate is the ratio of total new hires to the number of vacancies:

$$q(\theta_t) \equiv \frac{m_t}{v_t} = \overline{m}\theta_t^{-\mu}, \tag{9}$$

where $\theta_t = v_t/u_t$ is aggregate labor market tightness. The hiring rate describes the ease with which an open position can be filled for an individual firm. When the labor market is tight, θ_t is high, and the probability of filling a position is low. Similarly, from the perspective of a worker, we can define the job-finding rate:

$$p(\theta_t) \equiv \frac{m_t}{u_t} = \overline{m}\theta_t^{1-\mu}. \tag{10}$$

It is increasing in tightness since workers benefit from a low number of competitors and a large number of open positions.

The matching process is subject to congestion and thick-market externalities: An individual worker entering the labor market takes the probabilities of finding a job as given, while contributing to the deterioration of the job-finding chances of all other workers. The same logic applies to searching firms. At the same time, more search activity by one type of market participant, say firms, raises the success probability for the other type, the unemployed in this case.

The key components for the dynamics of employment are the choices of firms and workers. In the standard model, it is assumed that all workers participate in the labor market and aim to find work. We assume that a filled job produces a flow output of A_t with one worker, where A_t is aggregate productivity. Workers are remunerated with a wage w_t. The value of a filled job can then be written as

$$J_t = A_t - w_t + E_t\beta\left[(1-\rho)J_{t+1} + \rho V_{t+1}\right], \tag{11}$$

where β is a discount factor, and E_t is the expectation operator. While earning a net profit of $A_t - w_t$ in the current period, the job has either an expected value of $E_t J_{t+1}$, if it survives the job separation shock, or $E_t V_{t+1}$ if it is destroyed next period.

A free entry condition applies for firms. It posits that new vacancies are posted as long as the net benefit of a filled job exceeds the cost of posting a vacancy. Therefore, the value of a vacancy is $V_t = 0$ for all t, in equilibrium. Assuming vacancy posting costs of κ per period, the free entry condition implies that:

$$\frac{\kappa}{q(\theta_t)} = \beta E_t J_{t+1}. \tag{12}$$

The cost of posting a vacancy is κ times the expected duration of the vacancy $1/q(\theta_t)$ until it is filled. The benefit is the expected discounted value of a filled job when it starts producing next period. The two equations above can be combined to write the job creation condition (JCC), which is one of the key ingredients of the search and matching model:

$$\frac{\kappa}{q(\theta_t)} = \beta E_t \left[A_{t+1} - w_{t+1} + (1-\rho) \frac{\kappa}{q(\theta_{t+1})} \right]. \tag{13}$$

The left-hand side of this relationship is the effective cost of creating a vacancy, while the right-hand side is the expected benefit. The higher are net profits for the firm, the more willing it is to post a vacancy. In choosing the optimal number of vacancies, the firm also factors in the likelihood that a new hire will get separated at rate ρ and that there is an incentive to postpone hiring if future effective cost are small. The appearance of $\kappa/q(\theta_{t+1})$ indicates that, in equilibrium, the free entry condition is also expected to hold in future periods.

The employment accumulation equation and the JCC jointly determine the unemployment rate and vacancies for a given productivity and aggregate wage process. Wages themselves are the outcome of a bargaining protocol that splits the rents from an existing employment relationship. Thus, the wage in the search and matching model does not play an allocative role, because workers are randomly matched and cannot direct their search effort to any particular firm.[6] The sole basis for participating in the labor market is the expected return to search.

Yet, the model has a fundamental indeterminacy since any wage process is consistent with an employment allocation, as long as it is not too high to make the match unprofitable for the firm or too low to make search for a new job more profitable for the worker. What matters to the worker is the entire present value of being employed. Nevertheless, it has become common practice in the literature to let wages be determined by the Nash bargaining solution, which resolves the bilateral monopoly problem between a worker and a firm.

Workers and firms both derive surplus from the matching arrangement over which they bargain. The Nash bargaining solution is a particular time path for wages that shares the overall surplus generated by the match, based on their assumed relative

[6] A related class of labor market models assume that search is directed and allow firms to post wages in order to attract workers. Due to analytical complexity these models have not found widespread use in the analysis of labor market dynamics and policy. See, for example, Burdett and Mortensen (1998) and Mortensen (2003).

bargaining strength.[7] Nash bargaining results in a wage schedule of the form:

$$w_t = \eta \left(A_t + \kappa\theta_t\right) + (1-\eta)b. \tag{14}$$

Wage payments are a weighted average of the match productivity, which the worker can appropriate at a fraction η, and the outside option b, of which the firm obtains the portion $(1-\eta)$. Moreover, the presence of fixed vacancy posting costs leads to a hold-up problem where the worker extracts an additional $\eta\kappa\theta_t$ because it is costly for the firm to replace him. The wage equation can be substituted into the JCC to yield an expression with labor market tightness the only endogenous variable:

$$\frac{\kappa}{q(\theta_t)} = (1-\rho)\beta E_t\left[(1-\eta)\left(A_{t+1}-b\right) - \eta\kappa\theta_{t+1} + \frac{\kappa}{q(\theta_{t+1})}\right]. \tag{15}$$

We can now discuss the workings of the search and matching model by considering the following scenario. Suppose aggregate productivity A is expected to increase for all firms and to remain high for an extended period. This raises future expected revenue and profits for all firms, so that the right-hand side of the job creation condition increases. Firms will want to hire additional workers to take advantage of the higher productivity as long as the expected cost of hiring is lower than the expected benefit. But as all firms post more vacancies, labor market tightness θ increases and so does the expected duration of a posted vacancy. Equilibrium is restored when costs and benefits are again equalized.

How strongly hiring incentives respond to changes in A depends on how θ behaves and on how the last two terms in square brackets move relative to each other. Recall that $\eta\kappa\theta_{t+1}$ is the hold-up term from the wage setting equation, while $\kappa/q(\theta_{t+1})$ reflects the future value of a job. For example, if η is small, then the wage will not respond strongly to expected movements in θ. On the other hand, if η is large enough so that wages respond strongly to improvements in labor market conditions, it may well be that the wage movement largely offsets the incentives for hiring.[8]

We can now relate the economic incentives to post vacancies to actual labor market dynamics, as described by equation (7). Rewriting slightly, using the relationships

[7] The total surplus of a match is given by

$$S_t = J_t - V_t + W_t - U_t$$

where W_t and U_t are the present values of being employed and unemployed. These are similarly determined as the values J_t and V_t for firms.

[8] This aspect gave rise to the Shimer (2005) puzzle that spawned a large literature on labor market flows.

defined above, we find:

$$n_{t+1} = (1-\rho)n_t + p(\theta_t)(1-n_t). \tag{16}$$

The equation shows how rising labor market tightness θ raises employment as it increases the flow of workers out of unemployment $u_t = 1 - n_t$. The stronger $p(\theta_t)$ responds, the stronger unemployment declines in future periods. Since both unemployment and the job-finding rate have direct empirical counterparts, this captures the essence of the labor market search and matching model as a data-generating process.

Steady-State Analysis We now turn to a graphical analysis of the simple search and matching model. The exposition below follows Lubik (2013). The employment equation (7) and the JCC (15) can be rewritten in terms of the two endogenous variables u and v. Furthermore, it is often convenient for illustrative purposes to study the steady-state representation of the model. This avoids finding a dynamic solution, but nevertheless still conveys the underlying logic of the model.[9]

Using the definition of unemployment $u = 1 - n$, we can substitute out n and rearrange the steady-state employment equation as follows:

$$v = \left(\frac{\rho}{\overline{m}}\frac{1-u}{u}\right)^{\frac{1}{1-\mu}} u. \tag{17}$$

This curve describes an equilibrium locus of combinations of u and v such that inflows and outflows to the (un)employment pool are balanced. We note that it is downward-sloping in u-v space. This curve is often referred to as the Beveridge curve. Likewise, the steady-state representation of the JCC is:

$$1 - \beta(1-\rho)\frac{\kappa}{\overline{m}}\left(\frac{v}{u}\right)^{\xi} + \eta\kappa\frac{v}{u} = (1-\eta)(A-b), \tag{18}$$

where we have made use of the definition of the matching rate $q(\theta)$ and rearranged terms. Note that $v/u = \theta$ defines labor market tightness, so that the JCC uniquely determines the steady-state level $\overline{\theta}$ as a function of the model's parameters only. The job creation condition can thus be written as a linear function in u-v space: $v = \theta * u$,

[9] This practive is defended by Shimer (2005) on grounds that adjustment in the labor market is fast enough so that the economy does not spend too much time away from the steady state. This is doubtful in the presence of persistent shocks or a strong internal propagation mechanism as argued in Krause and Lubik (2010).

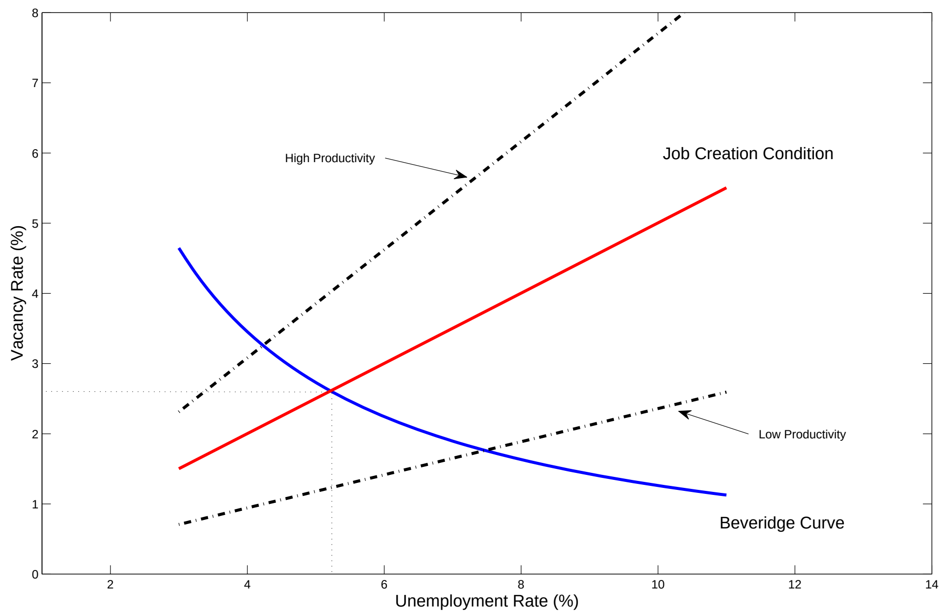

Figure 1: The Beveridge Curve and Equilibrium Vacancies and Unemployment

where shifts in the different structural parameters only affect the slope but not the intercept of the JCC.

Figure 1 depicts both curves in one diagram. The JCC is linear and upward-sloping, while the employment equation curves down. The JCC-curve captures the factors that matter for labor demand. The higher the pool of the unemployed from which a firm can hire, the more vacancies it will post for a given value of labor market tightness. The employment curve captures labor supply as it determines the evolution of the pool of the unemployed. When vacancy postings are high, many new hires are made, which depletes the pool of the unemployed. When vacancy postings are low, there are few transitions out of unemployment which therefore stays high. The intersection of both curves then determines the equilibrium in the labor market.

We now investigate which factors shift these curves.[10] We first note that aggregate productivity A, benefits b, bargaining power η, and vacancy posting costs κ only appear in the JCC but not in the Beveridge curve. The employment equation is unaffected by these parameters, whereas the separation rate ρ, the match elasticity

[10]For similar experiments in the continuous time version of the model, we refer to Pissarides (2000).

ξ, and the match efficiency $\overline{m}$ affect both curves. Increases in A raise the incentives to post vacancies and shift the JCC leftward in a parallel fashion. At a given unemployment rate, more vacancies are now being posted than is warranted by the initial productivity level. Consequently, the unemployment rate falls. This reduces the pool of available hires and makes it less likely that for a given vacancy posting the firm is successful in finding a worker. Since hiring costs are fixed and constant, firms post successively fewer vacancies (which results in fewer new hires), until the labor market settles at its new equilibrium level with higher vacancy postings and lower unemployment at a higher productivity level. The opposite logic applies for decreases in A. Shifts in productivity thus trace out a downward-sloping locus of equilibrium unemployment and vacancies, that is, the Beveridge curve. While the Beveridge curve coincides with the employment equation, it is actually generated by movements of the JCC along the latter.

We can apply similar reasoning to the other parameters in the JCC. Changes in benefits b have the exact opposite effect of productivity. Other things being equal, an increase in b increases the wage payments to workers (see equation (14)), which reduces the net profits to firms and thereby the willingness to post vacancies. The equilibrium outcome is a higher unemployment rate and lower vacancy postings. Increases in the bargaining power of workers have the same effect, as do increases in vacancy creation costs κ. Changes in these three parameters are often grouped under the label 'labor market policies' as they pertain to the structural characteristics of an economy. The simple search and matching model thus predicts that high unemployment benefits, high job creation costs, and a high degree of bargaining power on behalf of the workers each result in higher unemployment, an observation that has been confirmed in numerous empirical studies.

The search and matching model therefore describes the existence of equilibrium unemployment as the outcome of frictions both on the supply and demand side of the labor market. Being unemployed contains a measure of bad luck: The individual in question simply was not matched to a willing employer for reasons that an outside observer cannot ascertain. However, as we demonstrate above, this mechanism operates in an environment that is bounded by preferences and technology. Better employment outcomes arise when labor markets are more efficient in terms of the matching technology, when workers are more willing to supply labor, and firms are more willing to post vacancies. One question this raises, however, is whether this qualitative reasoning is empirically valid, that is, whether the search and matching model can capture salient labor market statistics.

4 The Quantitative Implications of the Search and Matching Model

The model equations are a data-generating process that produces a time-series representation for the labor market variables of interest. The model imposes theoretical structure on the data, without which we can at best derive correlation but will not be able to ascertain causation. The latter aspect is, of course, the crucial element in analyzing economic policies. We can compare the model's predictions for statistics of interest, such as the variance of the unemployment rate, to its observed counterpart in the data. Although our article largely proceeds along a theoretical perspective, it should be understood that there is always an empirical consideration in the background against which the success of a model should be evaluated. We proceed by first presenting some stylized labor market facts.

4.1 Stylized Macro Labor Facts

Figure 2 depicts monthly data on the unemployment rate and the vacancy rate over the post-WWII period.[11] The first panel shows a time series plot, while the second panel reports the same series as a scatter plot. There are some notable regularities. First, unemployment and vacancies strongly comove negatively. During recessions when unemployment is high, vacancy postings are low. This suggests an underlying mechanism that reduces the incentives to hire for firms in a downturn. An obvious candidate is aggregate productivity as a stand-in for the state of the business cycle, as we discussed in the theoretical framework above. When demand is weak, firms cannot hope to recoup the costs of hiring, hence the vacancy rate is low. Since there is not much new hiring, unemployment is high. A second observation is that both series exhibit a similar degree of variability. Moreover, the turning points in both series very

[11] The unemployment rate is measured as the total number of the unemployed 16 years and older as a percentage of the labor force. The vacancy rate is a composite of the Help-Wanted Index, collected by the Conference Board from the job openings in the 50 largest metropolitan areas, and the online job openings index as computed by Barnichon (2010). The reported numbers before 1995.1 are from the Conference Board, since 1995.1 from Barnichon (2010). It is an index number with 1987=100. We normalize this number by the labor force and report it in percentage terms.

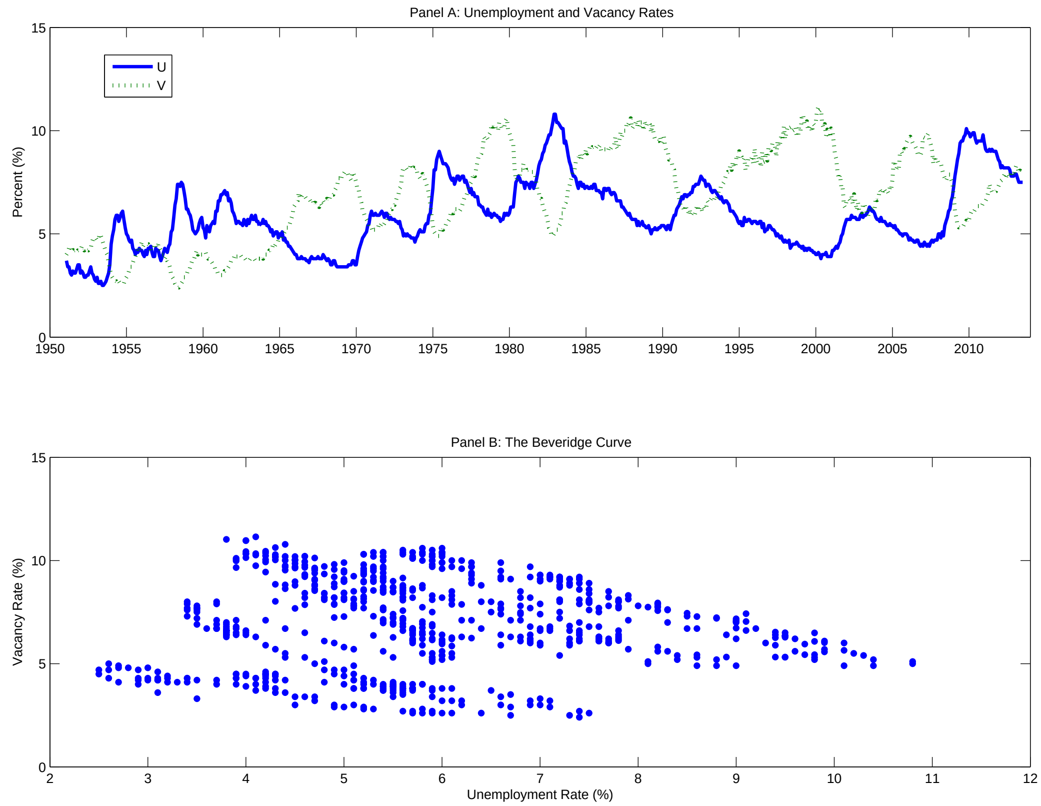

Figure 2: Unemployment and Vacancies in the U.S., January 1951 - June 2013.

much coincide, that is, when the vacancy rate is starting to fall, the unemployment starts to rise.

The first observation is a reflection of the well-known Beveridge curve. In the second panel of Figure 2 we show the relationship between unemployment and vacancies as a scatter plot, with the unemployment rate on the horizontal axis. A strong downward-sloping relationship emerges. The contemporaneous correlation of the series is -0.92 (see Table 1). Remarkably, this stylized fact is present over arbitrary sub-samples, at different sampling frequencies, and across countries. This statistical regularity therefore provides a benchmark against which to test any labor market theory. This observation is, in fact, at the heart of Shimer's (2005) puzzle, which we discuss in the next section.

Table 1 reports the standard deviations of the unemployment and vacancy rates and that of their ratios, namely labor market tightness $\theta_t = v_t/u_t$ over the full post-

World War II sample period. The statistics for the labor market variables are computed relative to that of U.S. GDP. All three variables are an order of magnitude more volatile than GDP, which is quite remarkable given that most macroeconomic variables are of the same order of magnitude. The failure of a standard search and matching model, when calibrated to commonly accepted parameter values, to capture these volatilities has been dubbed the Shimer puzzle, which has stimulated an enormous body of work.

Table 1: Stylized Facts and Model Simulations

	Data	Theoretical Models		
		Shimer (2005)	HM (2008)	Lubik (2009)
Standard Deviations:				
U.S. GDP	1.62	1.62	1.62	1.67
Unemployment	8.77	2.98	5.81	6.49
Vacancies	9.52	3.45	7.39	7.81
Tightness	13.24	3.82	5.22	4.36
Correlation:				
(Unemployment,Vacancies)	-0.92	-0.86	-0.67	-0.36

To summarize, a macroeconomic theory of the labor markets should be informed by some key stylized facts. First, measured unemployment is an equilibrium outcome in which workers cannot find employment despite potential employment opportunities as captured by the number of open vacancies. Second, unemployment and vacancies are highly negatively correlated (i.e., the Beveridge curve) and exhibit an exceptionally high degree of volatility.

4.2 Matching the Model to the Data

Macroeconomists working on labor markets have broadly applied two empirical strategies to match a model to the data. One strategy, calibration, assigns numerical values to the model's structural parameters and shock processes. The values can be obtained from extraneous observations or from long-run averages of model variables. The model is then simulated using the stochastic structure of the shock processes to produce time series of the variables of interests, from which the researcher can compute key statistics, such as the standard deviation of unemployment and vacancies. These statistics from the simulated model can then be compared to the statistics computed from actual data and the performance of the model along this dimension can be assessed at varying degrees of formality.

An alternative strategy is to estimate the model parameters directly using likelihood-based methods. The structural equations of the labor market model form a rational expectations model that can be solved for a reduced-form representation. The result is essentially a restricted time series model where the restrictions on the empirical model's coefficients are given by the relationships implied by the structural equations. Such a time-series model can be described by a likelihood function, which is a probability distribution for the model variables. The likelihood function can be estimated using actual data, and estimates for the structural parameters can be computed. The advantage of this method for assessing a model's performance is that it uses all information available to the researcher in a consistent manner. Moreover, it provides direct estimates of the structural parameters, including the degree of uncertainty about these estimates, and useful metrics about the overall fit of the model and thereby its validity as representation of the data. We will now discuss these two approaches in more detail as they apply to the quantitative analysis of the search and matching model.

Calibration Studies The calibration approach to deriving quantitative predictions from a theoretical model relies on assigning numerical values to the structural parameters. Values are typically being chosen by reference to existing studies, by preliminary empirical analysis or by specifying the value of an endogenous model variable and then adjusting parameters to hit that target value. Calibrating the search and matching model typically relies on a combination of these strategies. Given these parameter values, the model can then be formally evaluated by how well sample statistics from the calibrated and simulated model match those from the data.

The simple search and matching model can be calibrated along the following lines. The separation rate ρ can be inferred from transitions between employment to unemployment. Shimer (2005) reports a monthly rate 3.6%, so that $\rho = 0.036$. The match elasticity ξ can be estimated from the matching function using regression techniques. Petrongolo and Pissarides (2001) report values for $\xi \in [0.5, 0.7]$. The match efficiency can be inferred from the intercept of a matching function regression (see Lubik, 2013), which results in a value of $m = 0.8$, or it can be imputed by targeting an unemployment rate. The target value can be chosen as the sample mean over time, for instance. Depending on the sample period, and whether one takes a broader view of unemployed searchers as incorporating those outside of the labor force, the exact value of m can therefore vary for different calibration exercises.

The bargaining weight for the worker has to lie between 0 and 1, but there is no consensus on how to pin down the value for η. An agnostic approach sets $\eta = 0.5$. Some researchers calibrate η to match the volatility of wages in the data, yet another alternative sets $\eta = \xi$, which imposes the so-called Hosios-condition for social efficiency. A key parameter for the behavior of the model is the benefit level that workers receive while unemployed. Shimer (2005) chooses $b = 0.4$, which is based on actual benefits received over the course of the unemployment spell as a percentage of the previous wage, i.e., the so-called replacement ratio. An alternative calibration strategy is based on Hagedorn and Manovskii (2008) who emphasize that, while unemployed, workers can engage in home production, take on work in the shadow economy, or simply enjoy leisure. They calibrate b as high as a 90% replacement rate. In estimation exercises (discussed below), this parameter is usually found to come inbetween these extremes. Finally, the vacancy cost parameter is often calibrated by imposing an overall cost of hiring relative to GDP of $1-2\%$. The discount factor can be set at $\beta = 0.99$, while the productivity level can be normalized to $A = 1$, without loss of generality.

For specifications of the stochastic shocks the simple search and matching model can be simulated to produce statistics of the variables of interest. Table 1 reports the results from two calibration exercises that highlight key aspects of the model's dynamic performance. The shocks are calibrated such that the volatility of model output matches exactly that in the data. The table shows the so-called Shimer puzzle. Under the benchmark calibration of Shimer (2005) with a low replacement ratio of $b = 0.4$, the model fails to reproduce the volatility of unemployment and vacancies, which are almost an order of magnitude more volatile than those in the data. The reason is that wages are too volatile in the model, so that any productivity-driven incentives for firms to post more vacancies are quickly whittled away by rising wages. The model does match, however, the highly negative correlation of unemployment and vacancies since productivity changes move those two variables in opposite directions on impact and over the adjustment path.

Hagedorn and Manovskii (2008) suggest an alternative calibration that can raise the volatility of unemployment. The key channel is the reactiveness of wages to productivity shocks. This can be made more sluggish by lowering the bargaining power of the worker ($\eta = 0.05$), which limits the extent to which workers can appropriate higher surplus. A second channel raises the outside option of the worker. Hagedorn and Manovskii calibrate a replacement ratio of above 90%, which makes workers more reluctant to accept new jobs. Table 1 shows the simulation results from this alternative calibration. Unemployment, vacancies, and tightness are now more volatile but do not quite match the statistics in the data, whereas the Beveridge curve correlation now declines markedly.

While this alternative calibration can resolve the Shimer puzzle qualitatively and to a large extent quantitatively, it does not provide a good answer whether the model fits the data overall well. As Table 1 shows, improvement in one direction coincides with worsening of another relationship. The simple moment-matching approach cannot easily resolve the trade-offs between these two targets. In order to do so, the researcher has to rely on an empirical strategy that takes all the model implications into account by means of a likelihood function.

Structural Estimation One of the first attempts to estimate the standard search and matching model using likelihood-based methods was Lubik (2009). He estimates a slightly richer version of the simple model detailed above by introducing monopolistically competitive firms and variable vacancy posting costs. The latter allows for a role of mark-up shocks in driving business cycles and thereby an analysis of demand-like sources of fluctuations as in New Keynesian monetary models. The model is estimated on four variables: unemployment, vacancies, wages, and output for U.S. data from 1964-2008. The structural model serves as a data-generating process for these actual observable variables. That is, the researcher assumes that the real data were produced by these economic relationships through the interaction of exogenous stochastic shocks and endogenous model dynamics. While the true, underlying relationships outside of the model - in the real world, so to speak - are undoubtedly more complex, the researcher assumes that model relationships are good enough approximations.

The advantage of this approach is that the theoretical model has a time-series representation that can be compared directly to the time-series properties of the data. This can be accomplished by deriving the likelihood function of the model and estimating it on the actual data.[12] This produces point estimates, including standard errors, of the structural model parameters, statistics that measure the empirical fit of the model overall and along specific dimensions, such as the volatility of individual variables, and an assessment of the contribution of individual shocks in explaining fluctuations. This analysis is predicated upon the restrictions imposed by the theoretical model that ensure identification of the parameters and shocks. The model can thus be used for policy analysis. The disadvantage is that the underlying model, that is, the assumed data-generating process is incorrectly specified. This misspecification is likely to result in incorrect inference. Naturally, this conundrum cannot be fully resolved without knowing the actual true model, but it serves as a useful reminder when interpreting empirical results.

Lubik introduces four shocks into the search and matching model: a productivity shock, a shock affecting match efficiency, a shock to mark-ups, and a preference shock affecting the disutility of working. He finds that variations in match efficiency are by far the main driver of movements in the unemployment rate and to a lesser extent of vacancies. This is consistent with the notion of shifts in the Beveridge curve that we discuss further below (see also Lubik, 2013). Aggregate output is driven by productivity, which in this framework is a catch-all for the overall level of economic activity. Wage movements, on the other hand, are jointly explained by all shocks combined. What this finding leads up to is the idea that movements of labor market variables are largely decoupled from the rest of the economy, given the overall state of the business cycle.

This structural estimation exercise shows that the Shimer puzzle does not appear in the overall context of the richer model economy that is buffeted by a bevy of shocks. That is, the Shimer puzzle is more properly understood as a conditional statement of the relationship between labor productivity in vacancies. In an environment with various shocks it disappears. Lubik's results also show the calibration of Hagedorn and Manovskii (2008), which we discussed above is not strictly necessary to capture the business cycle facts. One concern that can be levied against this approach is that the nature of these driving shocks is largely unexplained. While the productivity

[12]Further discussion and references on how to perform these empirical exercises can be found in Lubik (2009).

shock can be tied to observable labor productivity, shocks to math efficiency are harder to rationalize. In the end, the shocks in structural models can be interpreted as standing in for deviations from optimality. We discuss this aspect below in the context of shifts in the Beveridge curve.

4.3 Labor Market Policies and Institutions

The search and matching approach has become the main framework for analyzing employment outcomes and the effects of labor market policies. We now discuss two recent strands of work in this vein. The first is an analysis of the German labor market reforms in the early 2000s, which is the most recent example of a large-scale policy effort to alleviate adverse labor market outcomes in an advanced economy. The second example is an analysis of potential structural shifts in the U.S. labor market following the Great Recession as seen in a shift in the Beveridge curve.

Labor Market Reforms In a broad brush of economic reforms, the German government introduced measures to increase the efficiency of the job finding process in the early 2000s, generally referred to as the Hartz IV reforms. There was rising concern that the German labor market had ground to a halt, with unemployment standing at 10 percent, and that long-term unemployment had become entrenched following the German unification. The central piece of reform, introduced in 2005, was the reduction in the level of unemployment benefits and in the duration of entitlements. Before Hartz IV, workers could earn unemployment benefit as a percentage of their previous income. The benefits would be paid out with an essentially unlimited duration. There was only a mild reduction after a duration of one to two years. Social welfare was mainly in place to support those who had never worked enough to build up entitlements for unemployment benefits.

The Hartz IV reforms implemented a reduction in unemployment benefit levels and duration. After the expiration of benefits typically within a year, recipients would transition to the social welfare system. This is a means-tested program, complemented by subsidies for housing and basic needs, such as food allowances. The underlying rationale was that the new system would increase worker incentives to search for and accept job offers, as well as reducing the costs of the welfare state. At the same time, there were reforms on the labor demand side that made it easier to recruit and lay off workers, such as an increased emphasis on probationary periods and short-term contracts.

The study by Krause and Uhlig (2012) aims to understand the effects of the German labor market reforms within the context of a search and matching model. They focus on the interaction of unemployment benefits and workers' skills. Following the approach pioneered by Ljungqvist and Sargent (1998), Krause and Uhlig introduce a mechanism where skills accumulate on the job but depreciate when a worker is off the job or unemployed. Workers lose jobs after idiosyncratic productivity shocks that hit jobs. In combination with an unemployment insurance system where benefits depend on previously earned income, skill loss during unemployment in their model makes workers less likely to accept job offers that align with their depreciated skills. The reason is that the worker's perceived reservation wage is still tied to the skill level before the unemployment spell. A new job offer will therefore be less attractive than continuing to receive relatively high benefits. Decoupling benefits from previous income after about a year is then likely to raise unemployment outflows in this framework.

The model is a combination of the two-sided search and matching approach, where the matching function determines the number of contacts between searching workers and firms, and the one-sided optimal stopping approach of McCall (1979) where the productivity of a match, and thus the wage offer, is stochastic. If the present value of working is more attractive than continued search, workers accept a job. Consequently, the higher the benefit is based on previous income relative to the new wage offer, the lower is the job acceptance rate. We can illustrate the basic effects of the reforms qualitatively by reducing the unemployment benefit b. However, the simple model lacks the structure to represent the changed duration of those benefits and the details of the welfare system. As such, it would not be able to give quantitatively reliable predictions of the effects of the reforms.

In Krause and Uhlig's simulations, the German labor market reforms reduce the long-term unemployment rate by more than 2 percentage points in the long run. They also reduce unemployment duration for the unemployed since both job acceptance rates and search effort by firms (i.e., vacancies posted) increase. Interestingly, wages of workers do not fall with the seemingly less attractive outside option of unemployment, because labor market tightness improves in their favor, which raises job finding rates and the workers' bargaining position. A dynamic simulation also shows that the adjustment of the labor market to aggregate shocks (such as those driving the downturn during the Great Recession) accelerates. Overall, the model predicts that the effects of the reforms have had most of their impact after about three years.

Compared to other studies, Krause and Uhlig (2012) find relatively large effects of the reforms on labor market outcomes. On the other hand, Wälde and Launov (2013) find low effects, the reason being that they interpret changes in entitlements as effectively not reducing unemployment benefits. Krebs and Scheffel (2014) find that the reforms contributed to a reduction in unemployment over the long run of about 1.5 percentage points. Overall, the various studies establish a benchmark as to how large the effects of such reforms possibly are.[13] They can thereby guide the development and assessment of changes to labor market institutions and provide input to policy decisions.

Mismatch in the Labor Market The effect of the Great Recession on the functioning and structure of the U.S. labor market has been the focus of intense research. One key issue is whether the sharp increase of the unemployment rate since 2008 and its persistence are due to cyclical factors, such as the weak overall economic activity, or whether structural changes have taken place that are likely to have raised the natural rate of unemployment. The issue was also the focus of debate among monetary policymakers, since the appropriate degree of monetary stimulus depends on how large the gap between cyclical and structural unemployment is.

In particular, Kocherlakota (2010) advanced the hypothesis that there was mismatch in the labor market in the sense that the open positions created by firms ready to expand production cannot be easily filled from the available pool of workers since their skills profile or the sectoral composition of U.S. production have changed over the course of the Great Recession. That is, mismatch may arise from workers and firms that search in different locations or because the skill composition of unemployed workers differs from that demanded by firms. Hence, the unemployed face more difficulty of being matched to vacancies. The implication for monetary policy is that it is of not much help to reduce the unemployment rate since the latter is structural in nature and will have to be addressed by labor market reforms of the type instituted in Germany.

The mismatch hypothesis is closely tied to the behavior of the Beveridge curve over

[13] Other related studies that use similar frameworks are Nakajima (2012) on the extension of unemployment benefits in the U.S., and Balleer et al. (2014) on short-time work subsidies during the Great Recession.

the course of the Great Recession. Figure 3 depicts the unemployment and vacancy rates over the 2000s until 2011. A regression line is fitted to the data up until the onset of the downturn. We see that the data points cluster closely around this stable curve until the middle of 2008 when the rise in the unemployment rate and the fall in vacancy postings accelerated. At the height of the recession, unemployment stayed stable at a high rate for several months (around 10%), while vacancy postings rose. Under normal circumstances, unemployment and vacancies move together along a stable Beveridge curve, so that higher vacancies are soon followed by falling unemployment. This did happen eventually, as depicted in Figure 3, but in a manner that suggests a parallel shift in the Beveridge curve. At every level of the unemployment rate, more vacancies postings are needed to generate the same number of hires and to reduce the unemployment rate.

Within the confines of the search and matching model presented above, this shift in the Beveridge curve can be captured by a decrease in the parameter $\overline{m}$, which represents the efficiency of the matching function. Structural breaks in match efficiency can explain the parallel movement in the Beveridge curve, whereas movements along the curve reflect the state of the business cycle as captured by sideways movements in the job creation condition (see Figure 1). Blanchard and Diamond (1989) argued that rapidly rising vacancy postings and slow declines in the unemployment rate follow the typical pattern of labor market adjustment over the business cycle, which they labeled a 'counterclockwise loop' as the vacancy-unemployment combinations would return to their initial location. What stimulated academic and policy debates is that the Great Recession was the larger shift than usual and that the adjustment process appears to be still ongoing.[14]

The argument that the noticeable shift in the Beveridge curve is cause by a structural break in match efficiency is made forcefully by Barlevy (2011) and Lubik (2013), based on, respectively, a calibration and a structural estimation of the simple search and matching model using aggregate data. In a sense, this argument is statistical in nature but informed by theory. However, this line of analysis faces the issue - as is often the case in macroeconomics - that it is very difficult to distinguish between permanent changes, such as a structural break in match efficiency, and persistent, but temporary changes. In other words, the seeming apparent shift in the Beveridge curve

[14] This view is being disputed by Benati and Lubik (2014) who argue that the observed shifts are not statistically distinguishable. Moreover, the observed shift during the Great Recession was of similar magnitude than those in the recessions of 1970s and in the early 1980s.

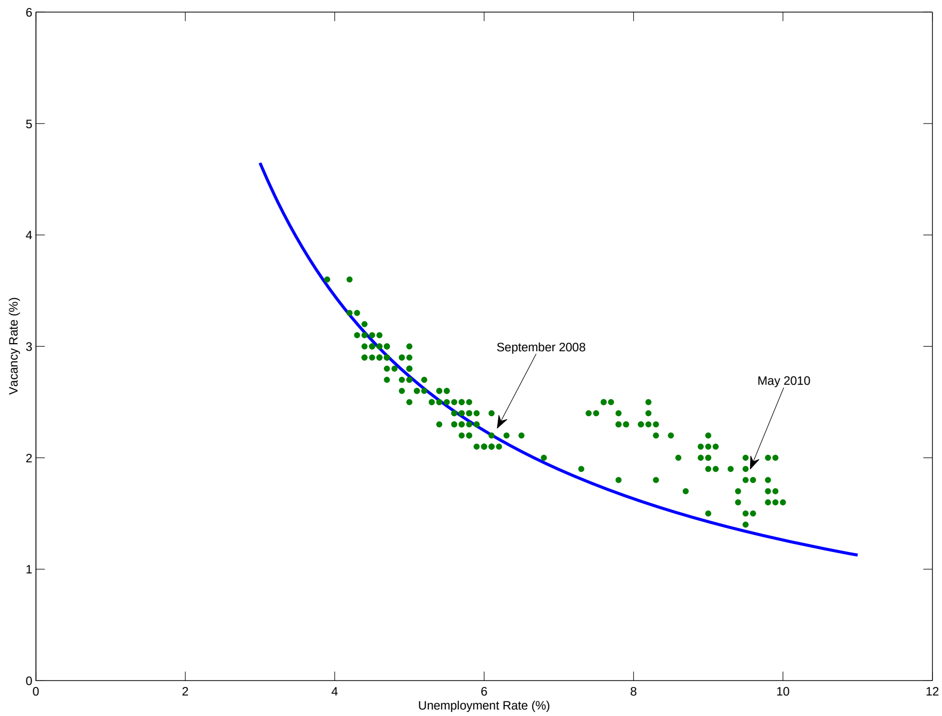

Figure 3: The Beveridge Curve in the Great Recession

is a prolonged counterclockwise loop. In the end, this question can be adjudicated only with the passing of time when more data become available.

An alternative approach to analyzing movements in the Beveridge curve is to use more disaggregated data and study the search outcomes of firms and workers at a more detailed sectoral or even individual level. Daly et al. (2012) summarize and discuss pertinent research along these lines. They highlight the effects of extended unemployment entitlements on job search incentives and increased uncertainty facing firms after the crisis as potential causes of mismatch. Using sector- and skill-specific matching technologies that they then aggregate into an economy-wide matching function, they show that a drop in match efficiency can generate higher unemployment due to a Beveridge curve shift. However, this can explain at best an increase in the natural rate of unemployment from about 5 percent before the crisis to about 6 percent afterward. Consequently, they argue that the increase in unemployment after the crisis and the slow reduction are to a large extent not explained by structural factors, but are caused by cyclical shocks that are likely to normalize in the medium run.

5 Monetary Policy and the Labor Market

One of the key issues in macroeconomics has always been the relationship between the nominal and the real side of the economy, perhaps the most important of which is the interaction between unemployment and inflation. The best-known statement of this kind is the Phillips curve, named after the New Zealand economist A.W. Phillips (1958), who found a statistical relationship between the level of unemployment and the rate of change of nominal wages. Over the years, the Phillips curve has come to be understood as a negative relationship between unemployment and price inflation over time. In much of the macroeconomic literature it is now considered a stylized fact and the object of much research.[15] This relationship between (un-)employment and inflation is also implicitly encapsulated in the "Dual Mandate" of the U.S. Federal Reserve, which is given the objective "to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates". We now discuss how the simple search and matching framework can help to rationalize this relation-

[15] A good overview of the various aspects of the Phillips curve from both the theoretical and empirical side can be found in a special issue of the Federal Reserve Bank of Richmond's Economic Quarterly "The New Keynesian Phillips Curve" (2008).

ship in a consistent manner.

A New Keynesian Search and Matching Model Modern monetary models are built around the concept of a New Keynesian Phillips curve (NKPC), which is derived from optimal intertemporal price-setting decisions of monopolistically competitive firms. These operate in an environment where each firm faces a downward-sloping demand curve that reflects consumers' preferences for variety in consumption goods. Since each firm in this environment produces a slightly differentiated product, consumers are reluctant to switch to a close but imperfect substitute unless the price differential is large. This feature allows firms to behave, within bounds, as a monopolist (hence, the moniker 'monopolistically competitive') and to choose the optimal, profit-maximizing price on the demand curve.

However, firms in a New Keynesian economy are also subject to a nominal rigidity in the form of impediments to free price adjustment. These can be modelled as Calvo-frictions, where only a fraction of firms are allowed to change prices at any period of time, or as explicit profit-reducing quadratic price adjustment costs as in Rotemberg (1982). In this framework, firms need to trade off the current and future benefits of adjusting their optimal price today in terms of extracting additional revenue along consumers' demand schedule against the cost of doing so. This implies that firms will not fully adjust prices contemporaneously when there are changes in nominal demand. Because of these nominal rigidities changes in nominal quantities will not be passed through one-for-one to (nominal) prices. It is through this mechanism that monetary policy can have real effects.

This relationship is captured mathematically by the NKPC, which is the optimal price-setting condition for monopolistically competitive firms under nominal rigidities. In its simplest (linearized) form, that sits nevertheless at the core of modern macroeconomic models that are used for policy analysis, it describes the interaction of current inflation, expected future inflation, and marginal cost, derived from first principles:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t. \tag{19}$$

β is the discount factor and $\kappa > 0$ is the NKPC coefficient, which is a composite of underlying, structural model parameters. mc_t is the marginal cost of firm, that is, the additional resources that a firm needs to expend to produce one more unit of output. Rising input costs mc_t lead to increases in inflation π_t. In the standard New

Keynesian model with perfectly competitive labor markets, marginal cost is directly proportional to unit labor cost, the wage relative to the marginal productivity of labor or, equivalently, the labor share.

However, in a labor market characterized by real rigidities, such as search and matching frictions, this equivalence need no longer hold. At the same time, the search and matching model provides a framework for how equilibrium unemployment moves over the business cycle. The challenge for economic modelers was how to integrate the labor market, characterized by the presence of real rigidities, with a model of intertemporal price movements and nominal rigidities in a consistent manner. This was accomplished in two important papers by Carl Walsh (2003, 2005) and two subsequent papers by Krause and Lubik (2007) and Trigari (2009).

The form of the NKPC with search and matching frictions in the labor market has been further worked out by Krause et al. (2008). Since marginal costs mc_t are equal to marginal revenue mr_t for a monopolistic price setter, the search and matching model can be essentially appended to the existing New Keynesian structure by noting that the job creation curve can be written as:

$$\frac{\kappa}{q(\theta_t)} = \beta E_t \left[x_t A_{t+1} - w_{t+1} + (1-\rho) \frac{\kappa}{q(\theta_{t+1})} \right], \tag{20}$$

where x_t is to be understood as per unit of output produced by a worker. When the equation is linearized analogous to the NKPC, this marginal revenue can be replaced by mc_t.[16]

The key insight is that in a search and matching model the cost of employing an additional worker is no longer just the wage but the value of the long-term relationship that the firm and worker enjoy. This depends on a host of factors: the bargained wage, the marginal revenue product of the worker, the ease with which a laid-off worker can be replaced, and the ease with which current employees can find other jobs. Krause et al. (2008) show that these influences interact in a complex manner. They furthermore demonstrate with a variety of empirical techniques that introducing search-and-matching considerations into the Phillips curve does not improve its empirical performance in a substantive manner. The reason is that the unobserved but imputed marginal cost series is not volatile enough to account for the volatility in inflation.[17] Nevertheless, the New Keynesian Phillips curve with search and matching

[16] Implicit in this formulation is that workers have linear marginal utility, as outlined in Krause and Lubik (2007).

[17] Similar results are obtained by Ravenna and Walsh (2008). The lack of empirical success of the

frictions has become the centerpiece of modern macroeconomic models that link the labor market to monetary policy in a consistent manner.

Implications for Policy The search and matching approach to the labor market is a laboratory not only for analyzing positive questions but also for studying normative issues, such as what policymakers should do when confronted with unemployment. Our discussion of these issues is framed by two aspects of our modeling approach. First, the model has explicit welfare-theoretic foundations in terms of utility defined over consumption (and possibly other variables). Unemployment is undesirable in this framework in that it is commensurate with a loss in income and hence consumption possibilities. The experience of unemployment is clearly a difficult one that any modeling of the kind in this chapter cannot do justice. However, couching it in a utility-theoretic framework allows the researcher to quantify the cost of unemployment in terms of lost consumption, which then can imply a ranking of policy measures. Furthermore, this allows the researcher to think in terms of trade-offs that policymakers should consider but in practice often do not.

The second, and related, aspect is that unemployment is a choice in the search and matching model, by which we mean it is the outcome of optimizing decisions taken by workers and firms given the environment, that is, under given labor market arrangements. If firms do not post enough vacancies, unemployment results. If workers prefer the outside option of enjoying leisure and collecting benefits, they remain unemployed. Job creation decisions are, however, framed by costs and benefits that labor market and economic policies can influence. Similarly, the unemployed are not likely to actively search for a job if a successful outcome is too unlikely or if the expected wage is below what they deem satisfactory. Again, the boundaries of the job search decision can be influenced by policy.

A choice-theoretic approach to modelling labor markets makes the inherent trade-offs transparent. In particular, such an approach is designed to address concerns arising from the Lucas-critique in that reduced-form relationships are not stable under policy changes. In other words, well-meaning policy measures based on empirical relationships are not a reliable guide and suffer from the law of unintended conse-

thus augmented Phillips curve can be traced to the feature that the labor market variables do not exhibit enough persistence since job posting and hiring are dominated by purely forward-looking considerations. Fujita and Ramey (2007) capture this persistence with state-dependent vacancy posting decisions.

quences. The simple view of the statistical relation between unemployment and inflation suggests that the choices for monetary policymakers, who are bound to deliver maximum employment and price stability, are clear. When unemployment is high and inflation is low, monetary policy can and should be expansionary. Yet, modern macroeconomics has shown that movements in real quantities matter for inflation dynamics only to the extent that they depart from their natural level, defined as the hypothetical level of, for instance, unemployment that would obtain in the absence of any distortions, such as impediments to free adjustment of nominal prices and wages.

The difference between actual and natural unemployment is often referred to as the "unemployment gap." It constitutes a measure of the degree of slack, or underutilization of resources, in the economy; a large and positive unemployment gap may constrain inflationary pressures. With a large pool of unemployed workers to hire from, wages are unlikely to increase, which therefore limits pricing pressures stemming from rising input costs via the mechanism embodied in the New Keynesian Phillips curve. In this setting, the best way to attain low unemployment *volatility* is to follow policy rules that promote price stability. How useful this argument is for policy decisions depends on how easy it is for policymakers to discern the level of the natural rate. The key insight of this line of thinking is, however, that the natural rate moves around with shocks and is not fixed. The question, then, is whether these fluctuations are "efficient" in the sense that stabilization policy should not or cannot effectively address them. On the other hand, fluctuations are "inefficient" if they are caused or amplified by forces that can be countermanded by policy.

The search and matching approach offers policy implications both for the short run and the long run. While stabilization considerations that can be addressed by monetary policy dominate in the short run, longer-run objectives have to be addressed by other means. These considerations are structural in nature in that they are tied to the primitives of the model (preferences and technologies). In the search and matching model, the steady-state unemployment rate is the rate of unemployment to which the dynamic system that describes the labor market would eventually gravitate toward in the absence of disturbances. This unemployment rate is therefore structural and outside the influence of short-run stabilization policies. However, the policy question that can be asked is what determines the long-run unemployment rate and whether it can be reduced by structural labor market policies.

6 Conclusion

In this chapter, we provide a snapshot of the currently dominant labor market model in macroeconomics and the issues that it helps to address. We focus on a simple framework that describes labor market outcomes, specifically the existence of equilibrium unemployment as determined by the rational choices of economic actors, namely workers and firms. The model captures real-world labor market interactions by means of a matching function and various rigidities that hinder the instantaneous labor market adjustment, which is emblematic of the neoclassical approach of perfect competition. A key aspect that we emphasize throughout the chapter is that theoretical models do not exist in a vacuum, but that they should be grounded in data and judged by their empirical success.

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