Online Appendix for: Searching for Hysteresis

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A The Data

Quarterly seasonally adjusted series for real GDP ('GDPC1: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate') are from the U.S. Department of Commerce, Bureau of Economic Analysis. Quarterly seasonally adjusted series for real chain-weighted investment, real chain-weighted consumption, and the chain-weighted consumption deflator have been computed based on the data found in Tables 1.1.6, 1.1.6B, 1.1.6C, and 1.1.6D of the National Income and Product Accounts. Whereas consumption pertains to non-durables and services, investment has been computed by chain-weighting the relevant series pertaining to durable goods; private investment in structures, equipment, and residential investment; Federal national defense and non-defense gross investment; and State and local gross investment. Inflation has been computed as the log-difference of the personal consumption expenditures (PCE) deflator. A quarterly seasonally adjusted series for total hours worked by all persons in the nonfarm business sector ('HOANBS: Nonfarm Business Sector: Hours of All Persons, Index 2009=100') is from the U.S. Department of Labor, Bureau of Labor Statistics.

A quarterly seasonally adjusted series for working age population (i.e., aged 15-64) has been constructed by linking the series from Francis and Ramey (2009) and that from FRED II, the Federal Reserve Bank of St. Louis' internet data portal ('LFWA64TTUSQ647S: Working Age Population, Aged 15-64, Noninstitutional, non-armed forces Population for the United States, Persons, Quarterly, Seasonally Adjusted'). Over the period of overlapping the two series are identical, which justifies their linking.

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Monthly series for the Federal Funds rate ('FEDFUNDS: Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted') and the 5-year government bond yield ('GS5: 5-Year Treasury Constant Maturity Rate') are from the Board of Governors of the Federal Reserve System. A monthly series for Wu and Xia's 'shadow rate' is from Cynthia Wu's website, at: https://sites.google.com/view/jingcynthiawu. All monthly interest rate series have been converted to the quarterly frequency by taking averages within the quarter.

We consider two sample periods, one excluding the Zero Lower Bound (ZLB) period, 1954Q3-2008Q4, and one including it, 1954Q3-2019Q4. We end the latter period in 2019Q4 in order to exclude the impact of COVID. As discussed in the main text of the paper, the starting date for the two samples is dictated by the fact that the Federal Funds rate is available since July 1954 (the 5-year goverment bond yield is available since April 1953).

B Unit Root and Cointegration Properties of the Data

B.1 Evidence from unit root tests

Table B.1 reports results from Elliot *et al.* (1996) unit root tests for the (log) levels of the series for the two sample periods 1954Q3-2008Q4 and 1954Q3-2019Q4, and for three alternative lag orders, $p = 2$, 4, and 6.

The null of a unit root cannot (near) uniformly be rejected for either sample for GDP, consumption, investment, long- and short-term nominal interest rates, hours, and the PCE deflator. On the other hand, as expected based on economic theory, it is uniformly rejected for either sample for the spread between long- and short-term rates.

Table B.1 Bootstrapped p-values for Elliot, Rothenberg, and Stock unit root tests[a]

	1954Q3-2008Q4			1954Q3-2019Q4		
Series	p=2	p=4	p=6	p=2	p=4	p=6
Log real GDP *per capita*	0.304	0.459	0.472	0.252	0.446	0.448
Log real consumption *per capita*	0.456	0.466	0.620	0.373	0.469	0.601
Log real investment *per capita*	0.344	0.255	0.171	0.235	0.172	0.110
Log total hours *per capita*	0.140	0.235	0.195	0.070	0.133	0.102
Long rate	0.501	0.429	0.545	0.603	0.544	0.668
Short rate	0.175	0.119	0.083	0.249	0.174	0.139
Log PCE deflator	0.233	0.104	0.150	0.993	0.977	0.936
PCE deflator inflation	0.095	0.098	0.179	0.006	0.025	0.085
Spread (long rate minus short rate)	0.000	0.000	0.000	0.000	0.001	0.000

[a] Based on 10,000 bootstrap replications. For details, see Appendix B.

Finally, turning to PCE deflator inflation, a unit root is strongly rejected for the period including the ZLB, whereas for the period 1954Q3-2008Q4 the rejection at the 10% level is weaker, with the p-value for p=6 being equal to 0.179. It is to be noticed, however, that for the full post-WWII sample period 1947Q2-2019Q4 rejection of a unit root is very strong, with the p-values equal to 0.000, 0.000, 0.002. In the paper we therefore work under the assumption that inflation is I(0), and that, conceptually in line with Benati (2008), the weak rejection of the null of a unit root for the period 1954Q3-2008Q4 is simply the figment of a comparatively short sample largely dominated by the experience of the Great Inflation.

B.2 Evidence from cointegration analysis

Turning to the cointegration properties of the data, basic economic theory suggests that, within the present context, we should expect at least three cointegration relationships: one between the short- and the long-term nominal interest rates, and two between GDP, consumption, and investment.

Table B.2*a* provides evidence that this is indeed the case. The table reports bootstrapped p-values[1] for Johansen's trace and maximum eigenvalue tests for three bivariate systems featuring GDP and consumption, GDP and investment, and a short- and a long-term nominal rate. For all systems evidence of cointegration is very strong, with the largest p-value across all systems being equal to just 0.0044.

Table B.2*a* Bootstrapped *p*-values for Johansen's cointegration tests[a]

	Trace tests:
Log real GDP *per capita* and	
log real consumption *per capita*[b]	3.0e-4
log real investment *per capita*[b]	0.0021
Federal Funds rate / Wu-Xia shadow rate and 5-year government bond yield[c]	0.0014
	Maximum eigenvalue tests:
Log real GDP *per capita* and	
log real consumption *per capita*[b]	3.0e-4
log real investment *per capita*[b]	0.0044
Federal Funds rate / Wu-Xia shadow rate and 5-year government bond yield[c]	0.0011

[a] Based on 10,000 bootstrap replications of Cavaliere *et al.*'s (2012) procedure. [b] 1948Q1-2019Q4. [c] 1954Q3-2019Q4.

[1] Bootstrapping has been implemented as in Cavaliere *et al.* (2012). Cavaliere *et al.* (2012), Benati (2015), and especially the Online Appendix of Benati *et al.* (2021) provide extensive Monte Carlo evidence on the excellent performance of this bootstrapping procedure.

Turning to the 7-variables systems, Table B.2*b* reports bootstrapped *p*-values for the corresponding tests for the two samples.[2] For the period 1954Q3-2008Q4 both the trace and the maximum eigenvalue tests point towards three cointegration vectors, whereas for the period 1954Q3-2019Q4 they point respectively towards four and three cointegration vectors. Accordingly, in the paper we proceed under the assumption that in both samples the 7-variables system features three cointegration vectors, and, as discussed there, we center the prior for the matrix encoding the cointegration vectors around the three cointegration relationships suggested by economic theory.

Table B.2*b* Bootstrapped *p*-values for Johansen's cointegration tests for the 7-variables systems[a]

	Trace tests of the null of no cointegration against the alternative of h or more cointegration vectors:				
Period	*h = 1*	*h = 2*	*h = 3*	*h = 4*	*h = 5*
1954Q3-2008Q4	0.040	0.034	0.052	0.600	
1954Q3-2019Q4	0.002	0.001	0.002	0.069	0.342
	Maximum eigenvalue tests of h versus h+1 cointegration vectors:				
	0 versus 1	*1 versus 2*	*2 versus 3*	*3 versus 4*	*4 versus 5*
1954Q3-2008Q4	0.142	0.057	0.030	0.782	
1954Q3-2019Q4	0.066	0.014	0.009	0.349	

[a] Based on 10,000 bootstrap replications of Cavaliere et al.'s (2012) procedure.

C The Real Business Cycle Model Used in the Monte Carlo Exercise of Section 5.2

Here follows a detailed description of the real business cycle (RBC) model we use as data-generation process (DGP) in the Monte Carlo exercise of Section 5. In essence, the model is that described in Galí's (2015) Chapter 2, augmented with habit-formation in consumption and a unit root in technology (and therefore in the natural level of output), and featuring the possible presence of hysteresis effects (as discussed below). Whenever possible we follow Galí's own notation. Households solve the following problem

$$U_0^* \equiv \underset{C_t, N_t}{\text{Max}}\; E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - bC_{t-1}) - \frac{N_t^{1+\phi}}{1+\phi} \right] \tag{D.1}$$

subject to

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t - T_t \tag{D.2}$$

[2] Again, bootstrapping has been implemented based on Cavaliere *et al.*'s (2012) methodology.

where C_t and N_t are real consumption and hours worked, respectively; $0<b<1$ is the habit-formation parameter; P_t is the price of consumption goods; B_t is the stock of nominal bonds; Q_t is the price, at time t, of a nominal bond paying 1 dollar at time $t+1$; W_t is the nominal wage; T_t are nominal lump-sum taxes; and the rest of the notation is obvious. The two first-order conditions (FOCs) are

$$N_t^{\phi} = \frac{W_t}{P_t} E_t \left[\frac{1}{C_t - bC_{t-1}} - \beta \frac{b}{C_{t+1} - bC_t} \right] \tag{D.3}$$

and

$$E_t \left[\frac{Q_t}{C_t - bC_{t-1}} \frac{P_{t+1}}{P_t} \right] = \beta E_t \left[\frac{1 + bQ_t}{C_{t+1} - bC_t} - \beta \frac{b}{C_{t+2} - bC_{t+1}} \right] \tag{D.4}$$

With no habit-formation (i.e., with b=0) the two FOCs collapse to those found in Galí (2015), i.e.

$$N_t^{\phi} = \frac{W_t}{P_t} \frac{1}{C_t} \quad \text{and} \quad 1 = \beta E_t \left[\frac{P_t C_t}{Q_t P_{t+1} C_{t+1}} \right] \tag{D.5}$$

As for the firms we exactly follow Galí (2015), with the only difference pertaining to the process for technology. Firms produce output (Y_t) via the production function $Y_t = A_t N_t^{1-\alpha}$, with $0<\alpha<1$ being the Cobb-Douglas parameter, A_t being technology, and the capital stock being constant and normalized to 1. Maximization of profits, $P_t Y_t - W_t N_t$, produces the FOC

$$\frac{W_t}{P_t} = A_t (1 - \alpha) N_t^{1-\alpha} \tag{D.6}$$

As for the logarithm of technology, $a_t = \ln(A_t)$, we postulate that it evolves according to

$$a_t = a_{t-1} + \epsilon_t^a + \delta \tilde{y}_{t-1} \tag{D.7}$$

with $\epsilon_t^a \sim N(0, \sigma_a^2)$, $\tilde{y}_t$ being the transitory component of output (to be discussed below), and $\delta \geq 0$ capturing the possible presence of hysteresis effects. If $\delta = 0$ there is no hysteresis, whereas if $\delta > 0$ positive (negative) transitory fluctuations of output cause subsequent permanent increases (decreases) in the level of technology. This specification, which is conceptually the same as the one used by Jordà et al. (2020), is designed to capture, in a very simple and stripped-down fashion, the notion that positive (negative) deviations of GDP from potential (here, deviations of output from its stochastic trend A_t) may have a positive (negative) impact on potential GDP itself. Although we could have considered more complex formalizations of the notion of hysteresis, we chose to use (D.7) because for our own purposes (i.e., performing Monte Carlo simulations) it is perfectly appropriate. Finally, since there is no investment, no government expenditure, and no foreign sector, $C_t = Y_t$.

Since a_t is I(1), log-linearization of the FOCs and of the production function requires the preliminary stationarizazion of the relevant variables. We define the stationarized variables

$$\tilde{Y}_t \equiv \frac{Y_t}{A_t} \quad \text{and} \quad \tilde{\Omega}_t \equiv \frac{\left(\frac{W_t}{P_t}\right)}{A_t}, \tag{D.8}$$

with $\tilde{y}_t$ and $\tilde{\omega}_t$ being the log-deviations of $\tilde{Y}_t$ and $\tilde{\Omega}_t$ from the steady-state, so that $\tilde{y}_t$ is the component of output that is driven by the transitory disturbances (discussed below). With $\tilde{Y}_t$ and $\tilde{\Omega}_t$ defined as in (D.8), the production function and the three FOCs can be trivially stationarized. Then, log-linearization of the stationarized production function and of the three stationarized FOCs produces the following four log-linear relationships characterizing the dynamics of the economy in a neighborhood of the steady-state,

$$\tilde{y}_t - (1-\alpha)n_t = 0 \tag{D.9}$$

$$\tilde{\omega}_t + \alpha n_t - v_t = 0 \tag{D.10}$$

$$-R_t + \pi_{t+1|t} + u_t + \phi n_t - \tilde{\omega}_t - \phi n_{t+1|t} + \tilde{\omega}_{t+1|t} = 0 \tag{D.11}$$

$$\phi n_t - \tilde{\omega}_t - e_t + \frac{1}{(1\text{-}b\beta)(1\text{-}b)}\left[(1+\beta b^2)\tilde{y}_t - b\beta\tilde{y}_{t+1|t} - b\tilde{y}_{t-1} + b\Delta a_t\right] = 0 \tag{D.12}$$

where $n_t \sim I(0)$ is the log-deviation of N_t from the steady-state, $\pi_t \equiv p_t\text{-}p_{t-1} = \ln(P_t)\text{-}\ln(P_{t-1})$ is inflation, e_t is a transitory demand disturbance, and v_t and u_t are two transitory supply disturbances, with

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{D.13}$$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \tag{D.14}$$

$$e_t = \rho_e e_{t-1} + \epsilon_t^e \tag{D.15}$$

with $\epsilon_t^v \sim N(0, \sigma_v^2)$, $\epsilon_t^u \sim N(0, \sigma_u^2)$, $\epsilon_t^e \sim N(0, \sigma_e^2)$, $|\rho_v, \rho_u, \rho_e| < 1$, and where the rest of the notation is obvious. The logarithm of technology, a_t, evolves according to (D.7). Monetary policy is characterized by a standard Taylor rule with smoothing,

$$R_t = \rho R_{t-1} + (1-\rho)\phi_\pi \pi_t \tag{D.16}$$

where the notation is obvious. Finally, we augment the model with an additive disturbance to the log of the production function, $z_t \sim N(0, \sigma_z^2)$, with σ_z=0.005.

By defining the state vector as

$$\xi_t = [R_t, \pi_t, \tilde{y}_t, n_t, \tilde{\omega}_t, \Delta a_t, v_t, u_t, e_t, \pi_{t+1|t}, \tilde{y}_{t+1|t}, n_{t+1|t}, \tilde{\omega}_{t+1|t}]' \tag{D.17}$$

and augmenting the system with the definition of the four rational expectations forecast errors

$$\pi_t = \pi_{t|t-1} + \eta_t^{\pi} \tag{D.18}$$

$$\tilde{y}_t = \tilde{y}_{t|t-1} + \eta_t^{\tilde{y}} \tag{D.19}$$

$$n_t = n_{t|t-1} + \eta_t^{n} \tag{D.20}$$

$$\tilde{\omega}_t = \tilde{\omega}_{t|t-1} + \eta_t^{\tilde{\omega}} \tag{D.21}$$

the system can be put into the 'Sims canonical form' (see Sims, 2002) and solved.

We calibrate the structural parameters as follows: β=0.99, α=1/3, b=0.8, ϕ=1, ρ=0.9, ϕ_π=1.5, and σ_a=0.007.[3] Of the three transitory AR(1) disturbances, two of them (v_t and u_t) are supply-side, and one (e_t) is demand-side. The AR parameters are calibrated to ρ_v=ρ_u=ρ_e=0.75, whereas the standard deviations of the disturbances' innovations ϵ_t^v, ϵ_t^u, and ϵ_t^e (all zero-mean, and normally distributed) are set to σ_u=0.001, σ_v=0.005 and σ_e=0.045. Based on this calibration the permanent technology shock (ϵ_t^a) explains exactly 1/3 of the forecast error variance (FEV) of log GDP on impact, and with δ=0 it explains slightly more than 96 percent of GDP's FEV 15 years ahead. These figures are broadly in line with the evidence produced by the structural VAR literature: for example, as for the fraction of the FEV of GDP explained by permanent shocks on impact see Table I.2 of Cochrane (1994). Further, based on these values of the structural parameters the demand-side disturbance ϵ_t^e is quite close to being the only driver of the transitory component of output, so that the identifying restrictions in Table 1 of the paper are, for practical purposes, correct. Finally, as for δ in the Monte Carlo exercise we consider a grid of values, from δ=0 (no hysteresis) to δ=0.1256, for which the technology shock ϵ_t^a and the demand-side shock ϵ_t^e both explain virtually half of the frequency-zero variance of output.

D Evidence Based on Stationary VARs

As mentioned in Section 3 in the main text of the paper, in spite of the results from unit root tests, the notion that interest rates and hours *per capita* are I(1) may be questioned on conceptual grounds. In this Appendix we therefore discuss evidence from stationary Bayesian VARs for either $Y_{1,t} = [\Delta y_t, R_t, r_t\text{-}R_t, \pi_t, h_t, CR_t^{y,c}, CR_t^{y,i}]'$ or $Y_{2,t} = [\Delta c_t, R_t, r_t\text{-}R_t, \pi_t, h_t, CR_t^{c,y}, CR_t^{c,i}]'$, where the notation is the same as in the main text; Δy_t and Δc_t are

[3] The value for σ_a is close to Watson's (1986, p. 60) estimate of the standard deviation of shocks to the stochastic trend of log real GDP (0.0057). The rationale is that, within the present context, a_t is the random-walk component of log real GDP.

the log-differences of real GDP and real consumption *per capita*; r_t-R_t is the spread between long- and short-term nominal interest rates; and $CR_t^{x,z}$ is the cointegration residual between x_t and z_t. To anticipate, these results are in line with those based on cointegrated SVARs in the main text of the paper, with hysteresis effects estimated to have been small to negligible for the sample including the ZLB period, and possibly *nil* for the sample excluding it.

Before proceeding it is worth clarifying the following issue. For the reasons discussed, e.g., by Cochrane (1994)—i.e., by the Permanent Income Hypothesis (PIH) consumption is very close to the unit root component of GDP—in what follows we pay special attention to the role played by H shocks in driving the long-horizon variation of consumption. Since the cointegrated system features both y_t and c_t, based on the estimated VECMs it is possible the explore the role played by H shocks in driving long-horizon variation in both variables. Based on the stationary VARs, on the other hand, this is not possible. In particular, based on the VAR for $Y_{1,t}$ it is only possible to explore this issue for y_t, since at the frequency zero c_t is obtained by rescaling y_t via the cointegration residual $CR_t^{y,c}$. This is why, within the present context, we are compelled to consider two alternative systems for $Y_{1,t}$ and $Y_{2,t}$. It is also a key reason why, overall, our preference goes to the evidence produced by the cointegrated SVARs.

For either $Y_{1,t}$ or $Y_{2,t}$ we estimate the following stationary VAR

$$Y_{i,t} = B_0 + B_1 Y_{i,t-1} + ... + B_p Y_{i,t-p} + u_t, \tag{D.1}$$

where the notation is obvious, and $i = 1, 2$. We estimate the Bayesian VARs as in Uhlig (1998, 2005). Specifically, we exactly follow Uhlig's approach in terms of both distributional assumptions (the distributions for the VAR's coefficients and its covariance matrix are postulated to belong to the Normal-Wishart family) and of priors. For estimation details the reader is therefore referred to either the Appendix of Uhlig (1998), or to Appendix B of Uhlig (2005).

We jointly impose the zero long-run restrictions, and the short- and long-run sign restrictions, based on the methodology proposed by Arias et al. (2018). We impose the short-run sign restrictions both on impact and for the subsequent either four or eight quarters. Since the two sets of results are qualitatively the same, in what follows we only report and discuss those based on imposing the restrictions for eight quarters, but the alternative set of results is available upon request. For each draw from the posterior distribution of the reduced-form VAR we consider 100 random rotation matrices that we draw based on Arias et al.'s (2018) algorithm. We set the number of Gibbs-sampling iterations in the algorithm to 10.

D.1 'Simple' Evidence from Stationary VARs

Figures A.10 and A.11 show the median and the 16-84 and 5-95 percentiles of the posterior distributions of the impulse-response functions (IRFs) to BQ and H shocks, and the fractions of forecast error variance (FEV) of the variables explained by the shocks, for the sample including the ZLB period, and without imposing restrictions on the long-run impact of H shocks on the price level. Figures A.12 and A.13 show results for the sample excluding the ZLB period. Finally, Figures A.14-A.17 show the corresponding set of results obtained by imposing restrictions on the long-run impact of H shocks on the price level. Exactly as for the results based on the cointegrated SVARs, a common feature that uniformly pertains to all Figures A.10-A.17 is a large extent of uncertainty.

The IRFs require little comment, since they are near-uniformly as expected (for GDP, consumption, investment, and the price level this is obviously the product of the restrictions that are being imposed). For example, at essentially all horizons the response of either the short rate, or the long-short spread to either shock is uniformly insignificant at the two, and often even at the one standard deviations level. Different from the evidence produced by cointegrated SVARs, the response of hours is estimated to be transitory for either shock. Focusing on the price level, the IRFs for the two samples obtained without imposing restrictions on the long-run impact of H shocks are in line with the corresponding evidence produced by cointegrated SVARs. In particular,

first, for the sample including the ZLB (Figure A.10) a non-negligible mass of the posterior distribution of the IRF to H shocks lies below zero. Further, at the frequency zero, the long-run impact of H shocks on the price level is estimated to be negative for 50.9 percent of the draws. As discussed in the main text, this suggests that the data provide some support to the notion, discussed in Section 2 (see the panel labelled as 'Hysteresis II' in Figure 1), that a positive (negative) H shock may counterintuitively have a negative (positive) long-run impact on the price level.

Second, for the sample excluding the ZLB (Figure A.12), at the 15 years horizon 69.6 percent of the draws are associated with a negative impact of H shocks on prices, whereas at the frequency zero the corresponding fraction is equal to 85.3 percent. As discussed in the main text, a possible interpretation is that in this sample H shocks are (virtually) absent, and that this result is simply the figment of the fact that our identifying restrictions impose upon the data their very existence by 'brute force' (see the discussion there). This explanation is compatible with the fact that, as we show in Section 5 in the main text, spuriously identifying hysteresis when the DGP features none by construction is, in fact, quite likely.

Table D.1 Fractions of frequency-zero variance of GDP and consumption explained by H shocks (median and 16-84 and 5-95 percentiles)

	Restricting the long-run impact of H shocks on the price level:	
	No	Yes
1954Q3-2019Q4		
GDP	0.325 [0.070 0.759] [0.013 0.944]	0.249 [0.038 0.740] [0.005 0.947]
consumption	0.280 [0.052 0.723] [0.011 0.929]	0.240 [0.033 0.735] [0.005 0.945]
1954Q3-2008Q4		
GDP	0.140 [0.024 0.474] [0.004 0.768]	0.025 [0.003 0.116] [0.000 0.267]
consumption	0.094 [0.015 0.333] [0.002 0.624]	0.024 [0.003 0.109] [0.000 0.249]

Turning to the frequency zero, Table D.1 reports the median and the 16-84 and 5-95 percentiles of the posterior distributions of the fractions of long-run variance of GDP and consumption explained by H shocks. Once again, estimates are characterized by a remarkable extent of uncertainty, which, as mentioned in the man text, is intrinsic to the investigation we are performing. Ignoring this issue, and focusing on the median estimates, the following facts emerge from Table D.1:

first, in line with the evidence from cointegrated SVARs, the estimates for consumption are most of the time smaller than those for GDP. The exception is the period excluding the ZLB period when restricting the long-run impact of H shocks on the price level, for which the estimates for GDP and consumption are virtually identical.

Second, focusing on consumption, which by the PIH and the evidence in Cochrane (1994) should be regarded as likely producing more reliable and informative results, evidence clearly suggests that estimates for the sample excluding the ZLB are dramatically lower than those for the full sample. E.g., the median estimates for the samples including and excluding the ZLB period are equal to 0.280 and 0.094, respectively, when not imposing restrictions on the long-run impact of H shocks on the price level, whereas they are equal to 0.240 and 0.024 when imposing such restrictions. This shows that for the sample excluding the ZLB period hysteresis effects are very small to negligible, and they only appear when also considering the years including the financial crisis and the Great Recession.

Third, restricting the long-run impact of H shocks on prices consistently produces smaller estimates of the fraction of the unit root of either GDP or consumption explained by H shocks.

Overall, the evidence produced by stationary VARs points towards a non-negligible extent of hysteresis, equal to 24 or 28 percent of the frequency-zero variance of GDP for the sample including the ZLB period. For the sample excluding the ZLB period estimates are significantly smaller.

D.2 Evidence From the Monte Carlo-Based Approach

Figure A.18 shows the median and the 16-84 and 5-95 percentiles of the Monte Carlo distributions of the Kolgomorov-Smirnov (KS) statistics for the two samples excluding and including the ZLB, respectively. Based on the stationary VARs featuring Δy_t and Δc_t we report individual statistics for GDP and consumption, respectively, whereas we eschew investment because, by construction, its frequency-zero behavior is (up to a scale factor) identical to that of y_t and c_t, respectively. Once again, evidence is characterized by a significant extent of uncertainty. Abstracting from this, and focusing on the median estimates, two facts emerge from the two figures:

first, for the sample excluding the ZLB period evidence uniformly suggests that the most likely value of the extent of hysteresis is zero. This is starkly apparent based on consumption, which, for the reasons discussed in the main text, should be regarded as the most reliable.

Second, for the full sample period evidence clearly points towards a non-negligible extent of hysteresis, equal to about 10 percent of the frequency-zero variance of GDP based on the VAR featuring Δy_t, and to nearly 30 percent based on the VAR featuring Δc_t.

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Figures for the Online Appendix

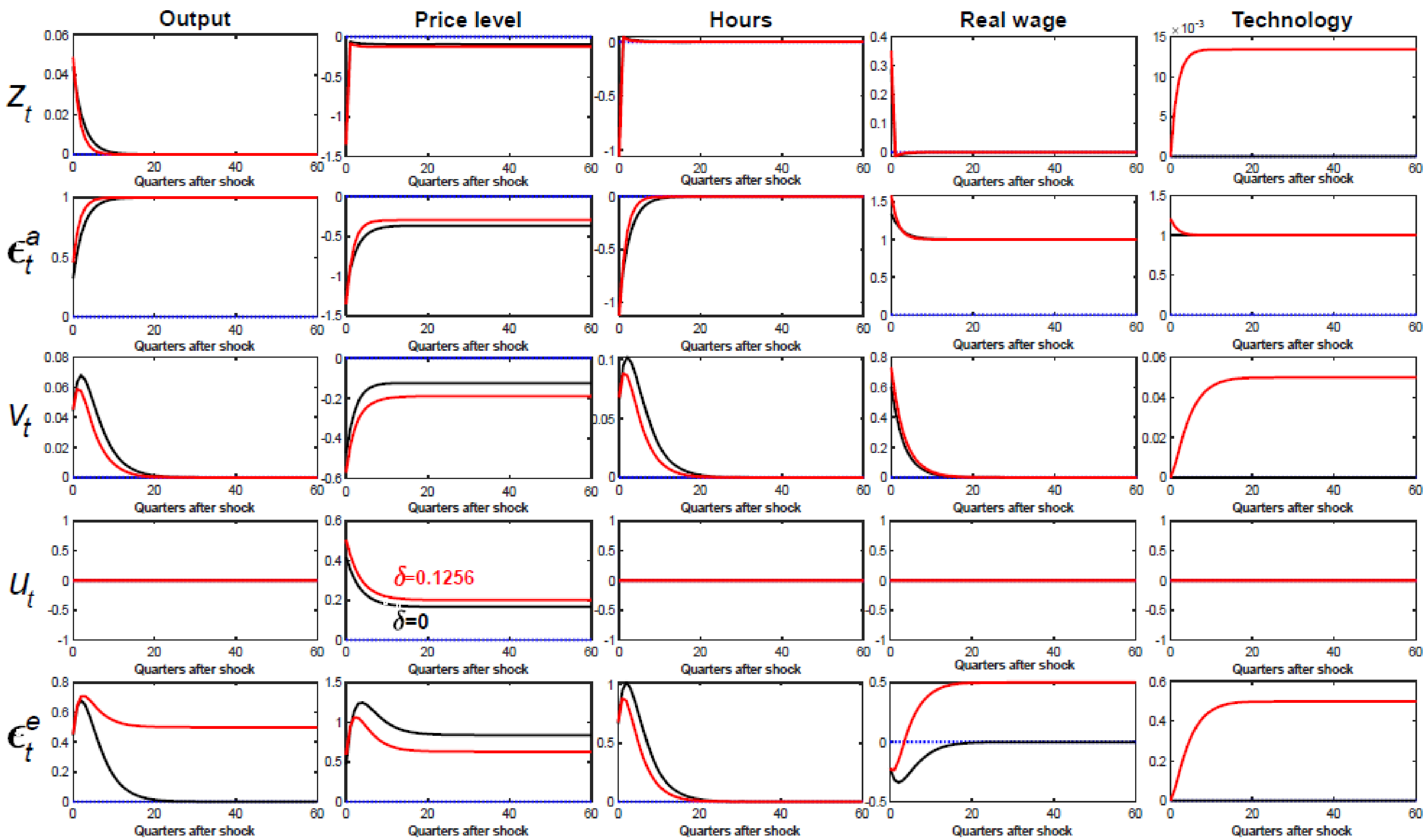

Figure A.1 Theoretical impulse-response functions for the RBC model

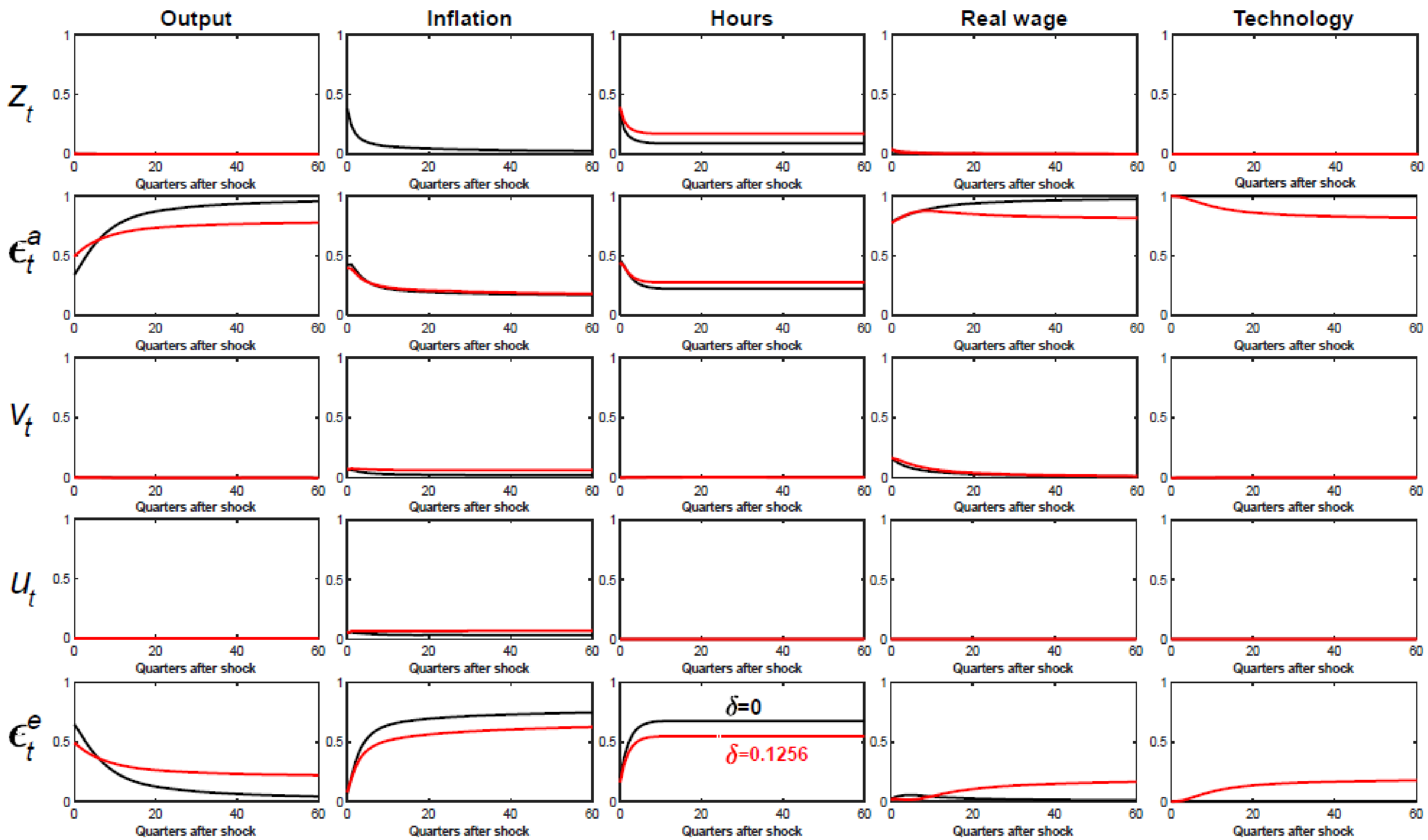

Figure A.2 Theoretical fractions of forecast error variance for the RBC model

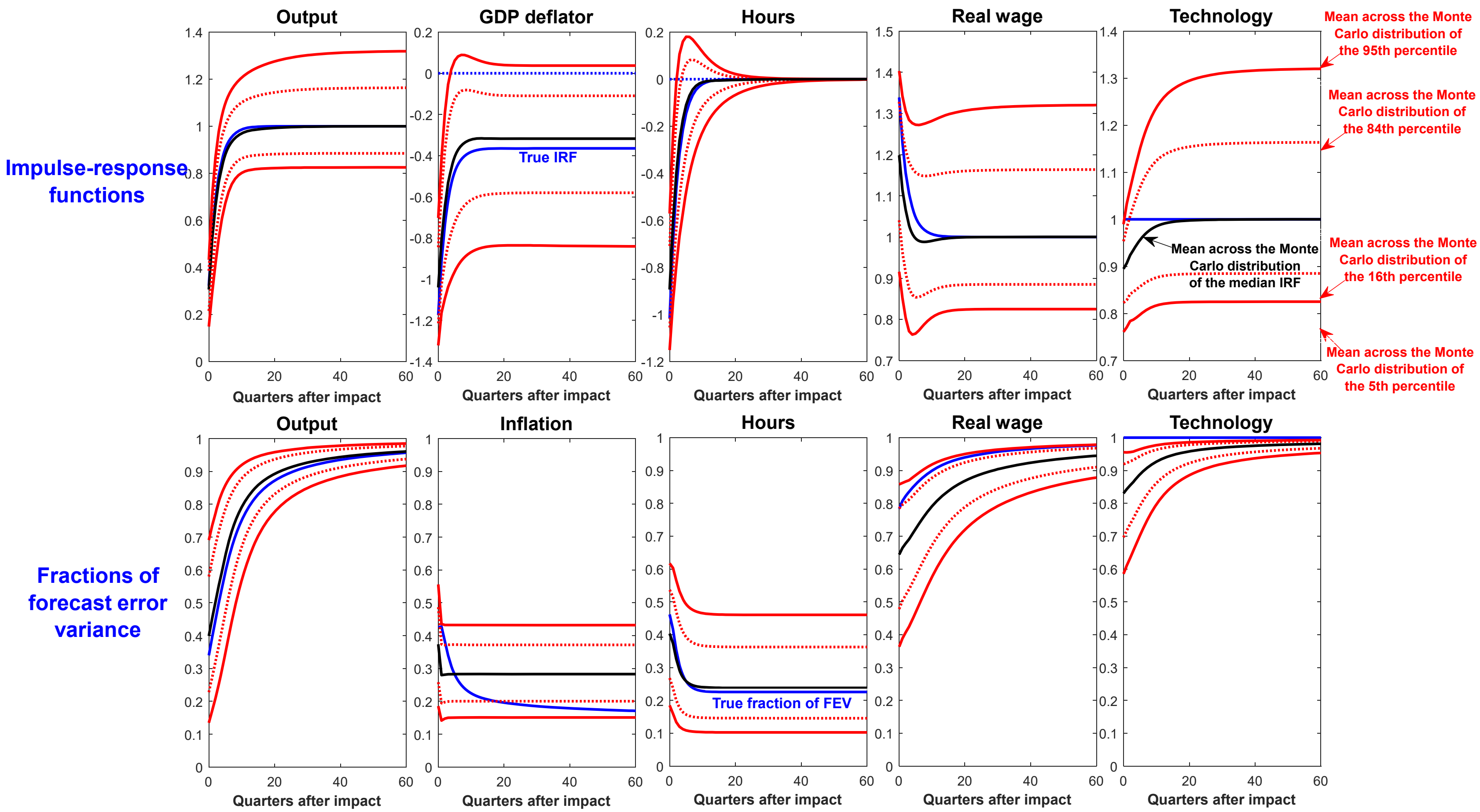

Figure A.3 Monte Carlo evidence on recovering the RBC model's true IRFs and fractions of forecast error variance with no hysteresis, conditional on the correct identification scheme

Evidence Based on the 7-Variables Bayesian Cointegrated VARs

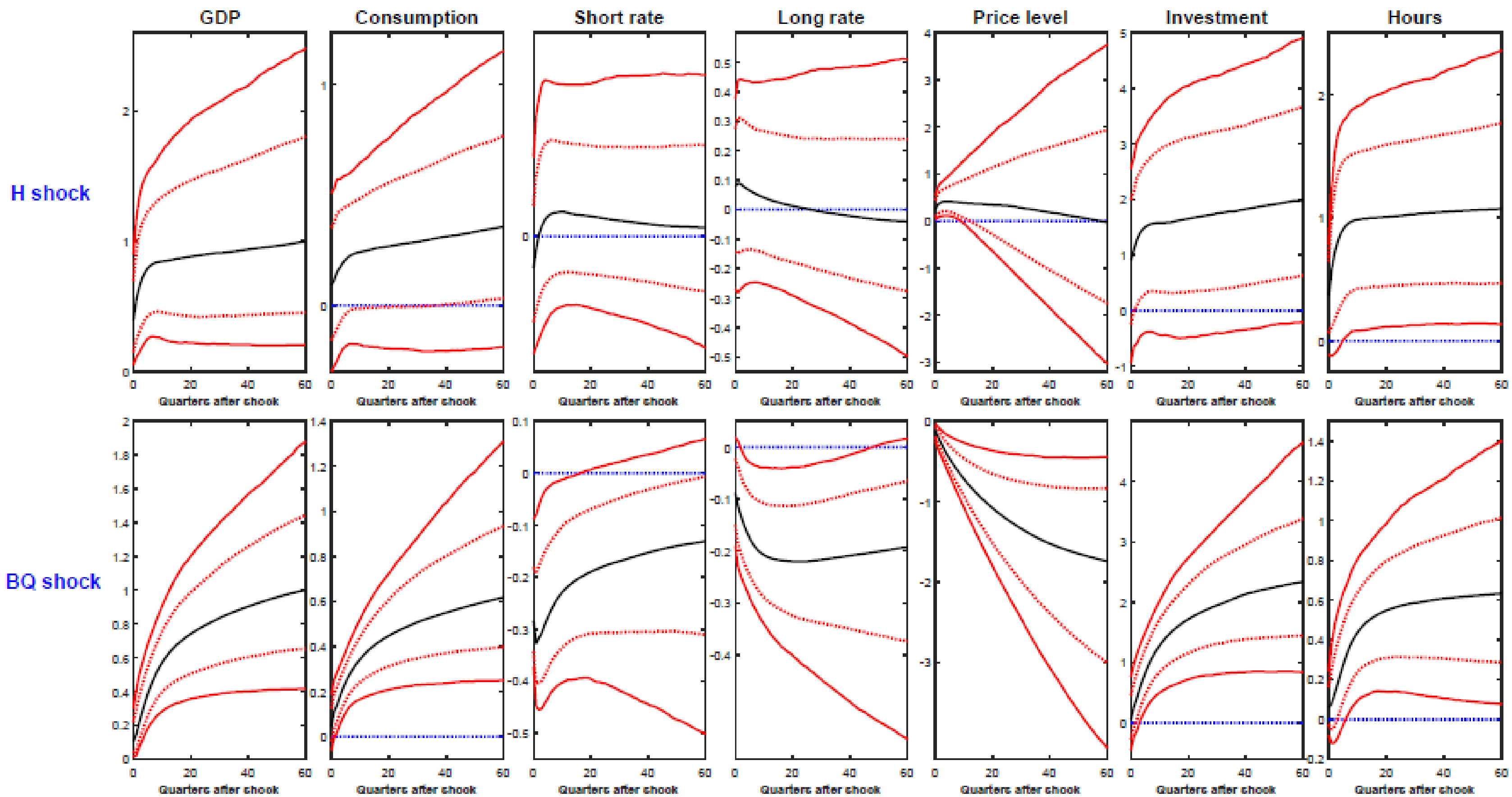

Figure A.4 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR (excluding the ZLB period)

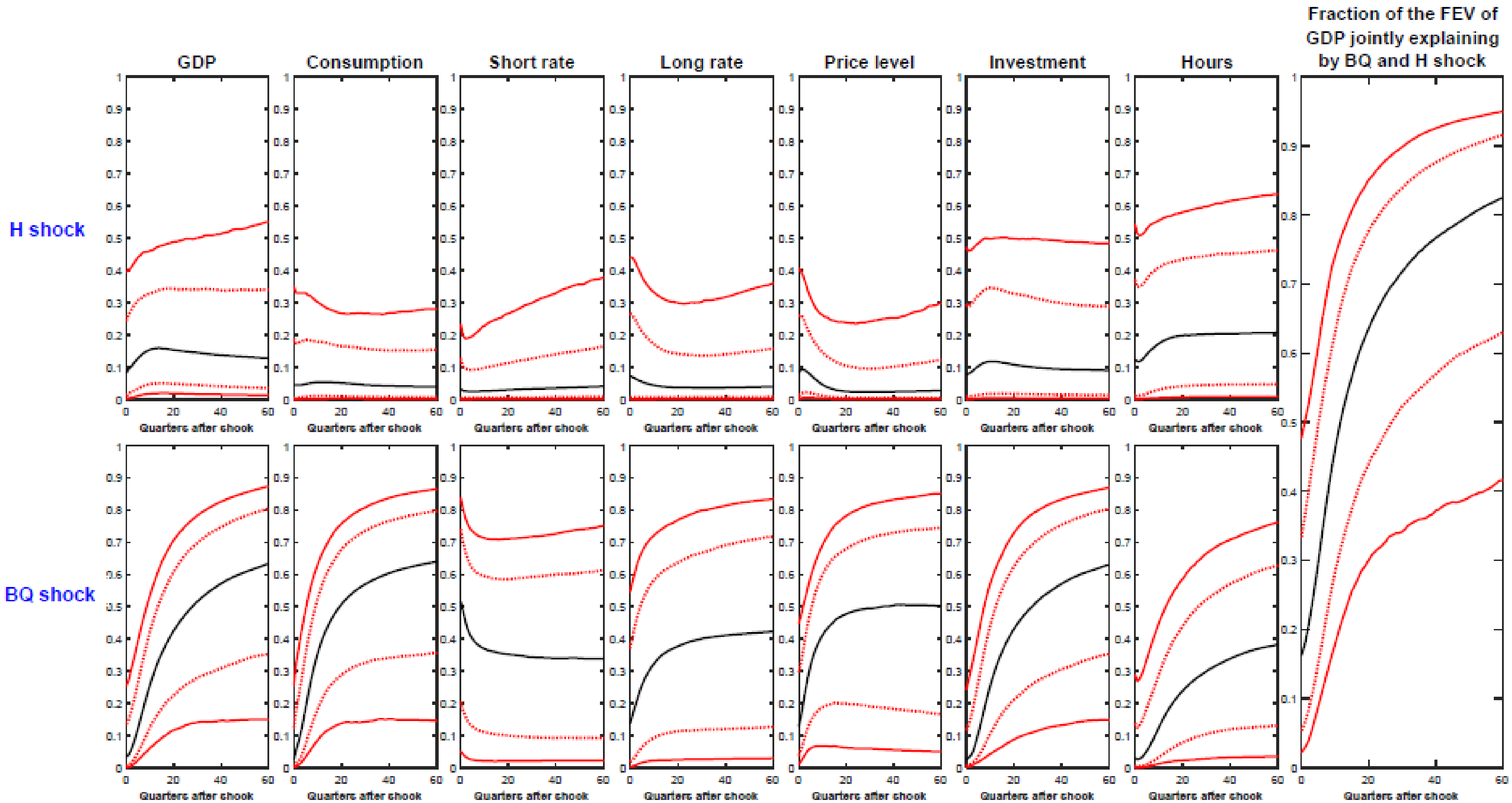

Figure A.5 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR (excluding the ZLB period)

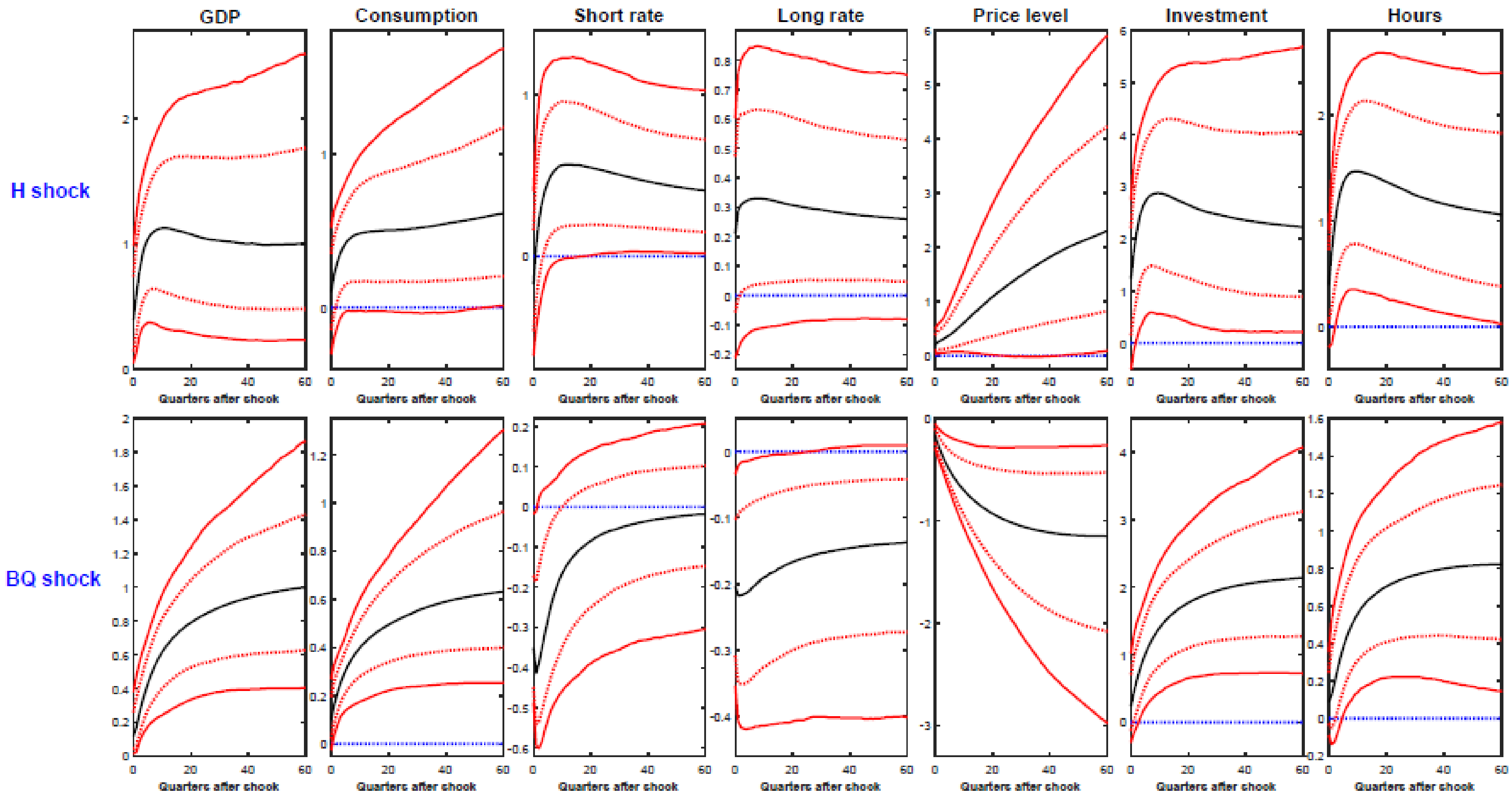

Figure A.6 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (including the ZLB)

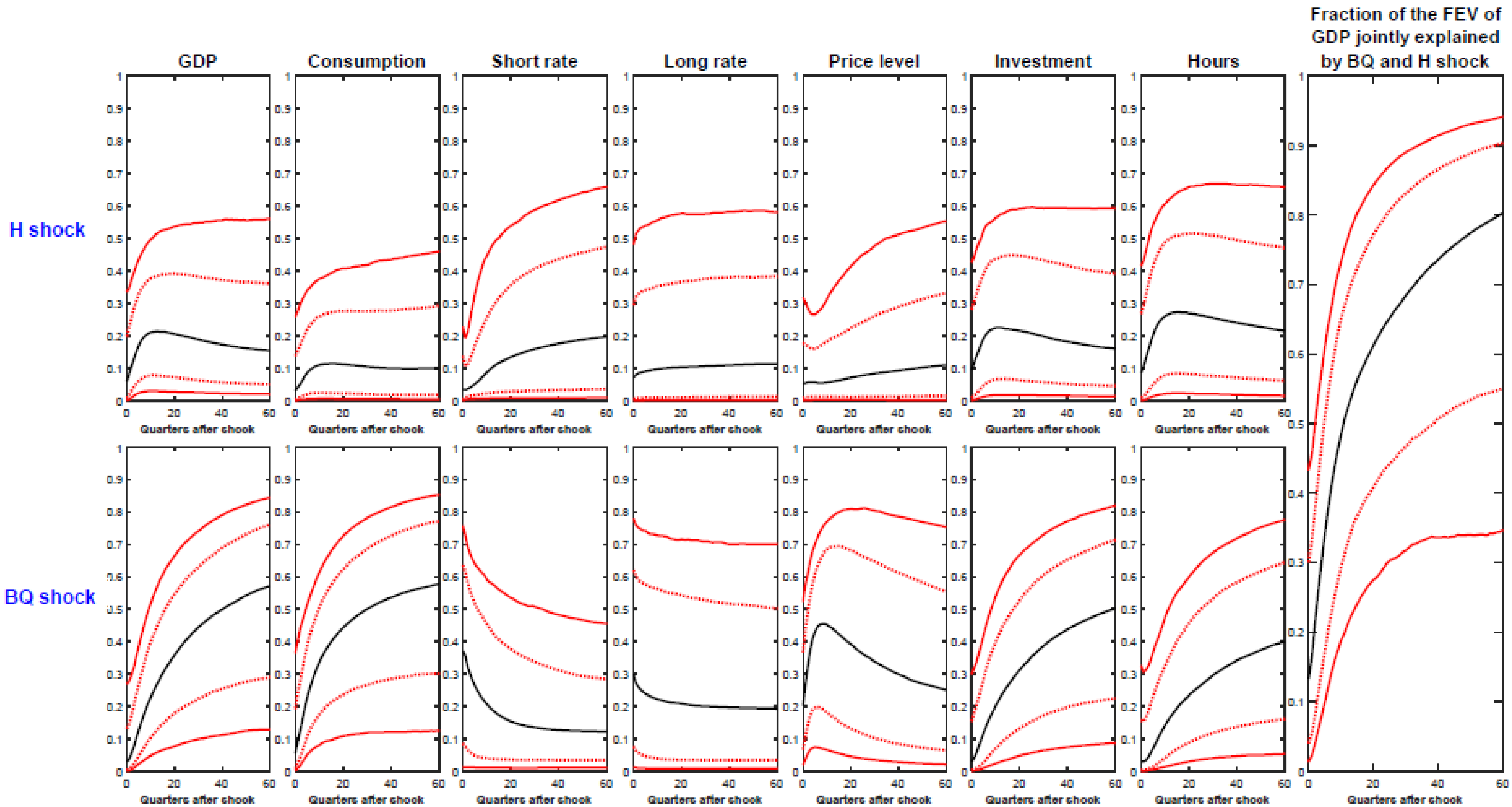

Figure A.7 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (including the ZLB)

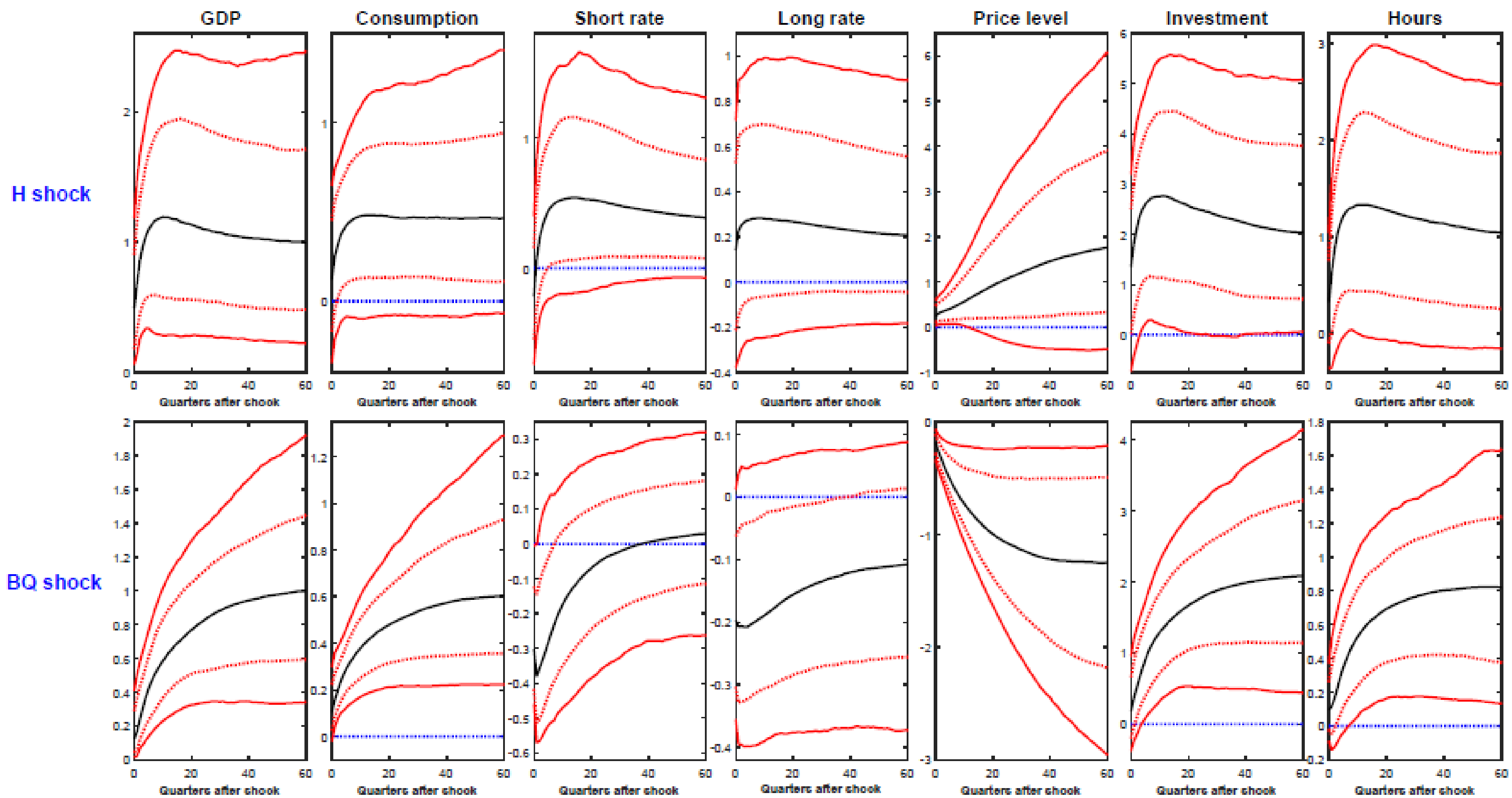

Figure A.8 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

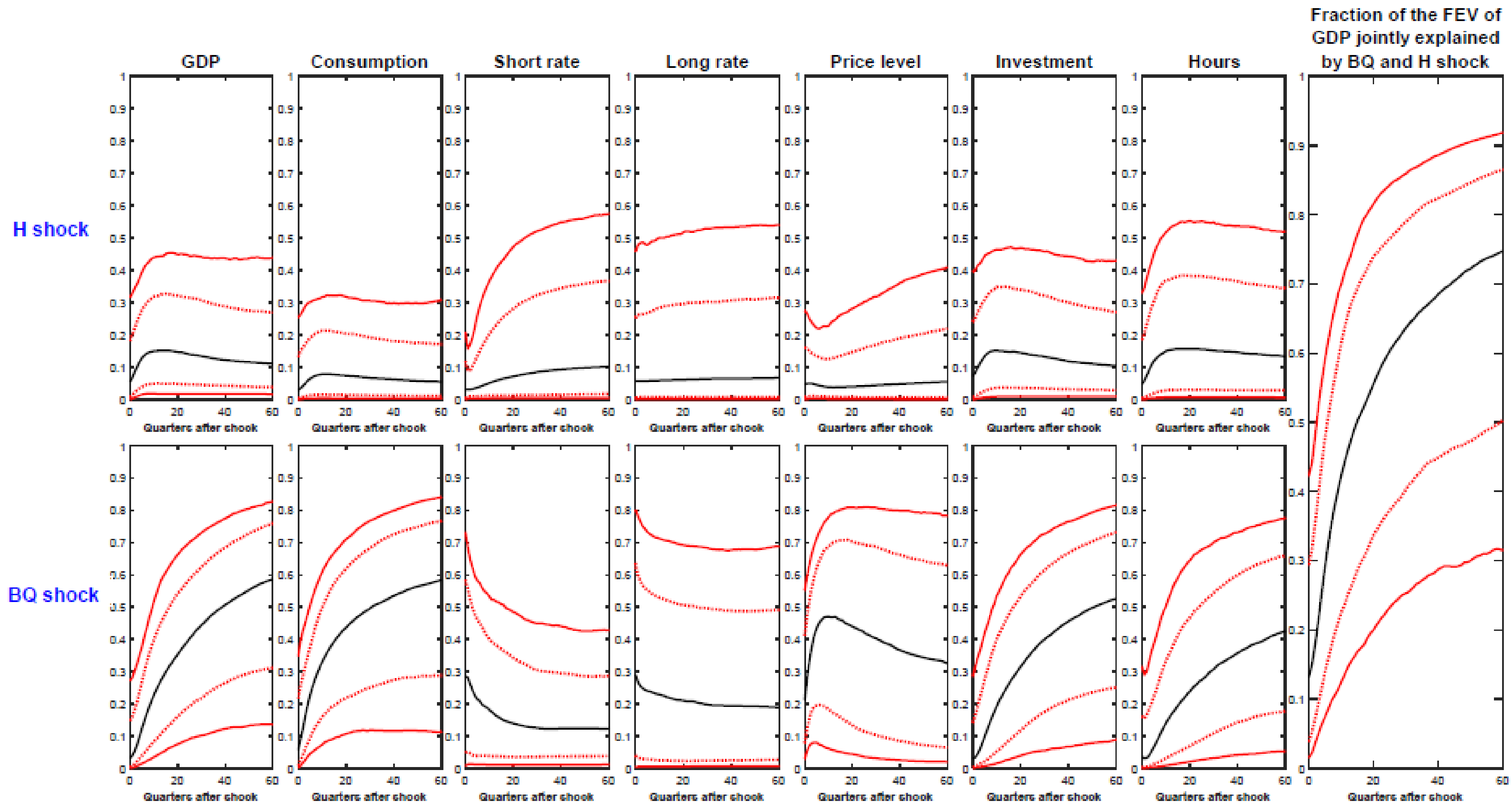

Figure A.9 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

Evidence Based on the 7-Variables Stationary Bayesian VARs

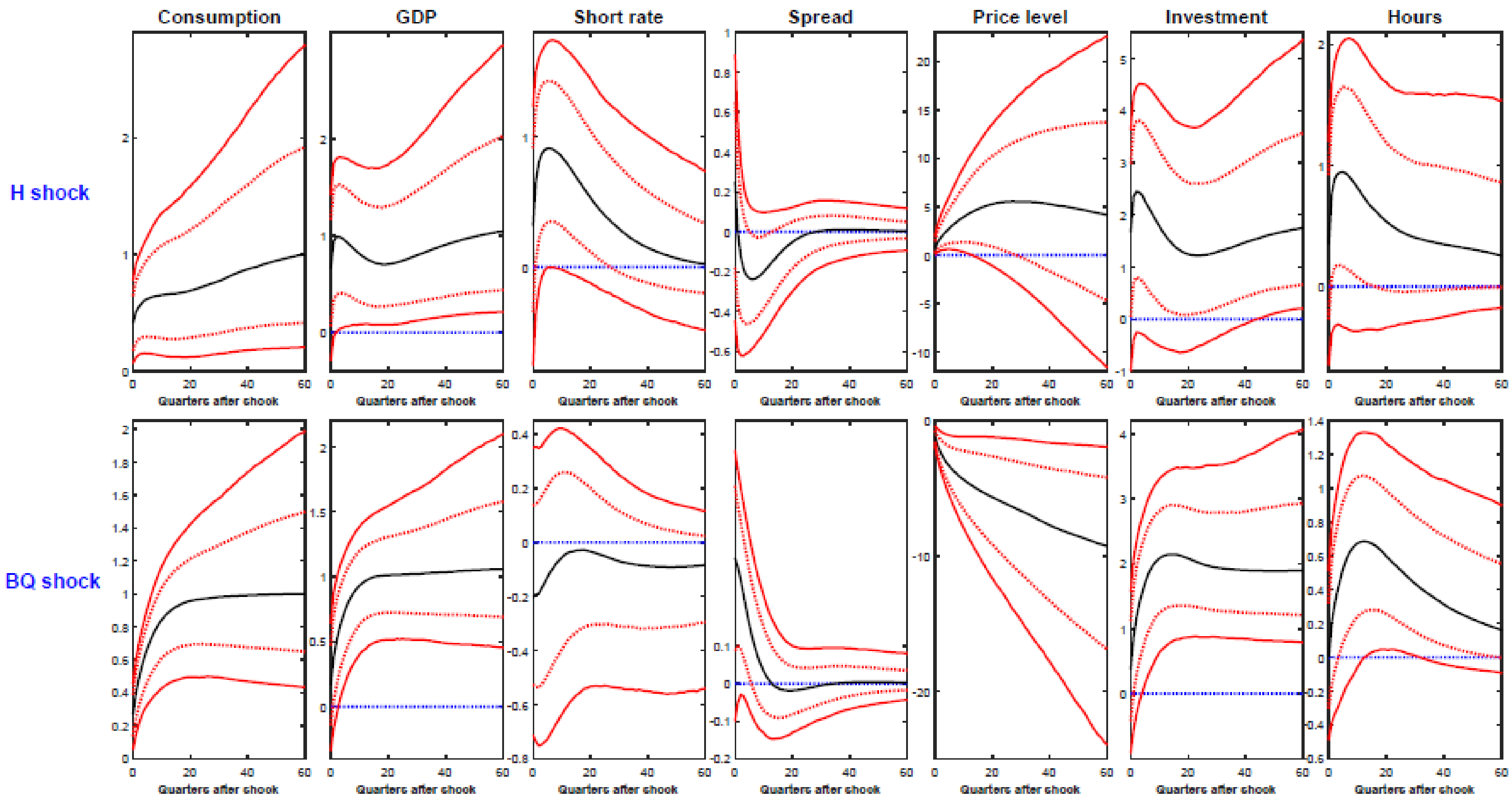

Figure A.10 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR (including the ZLB period)

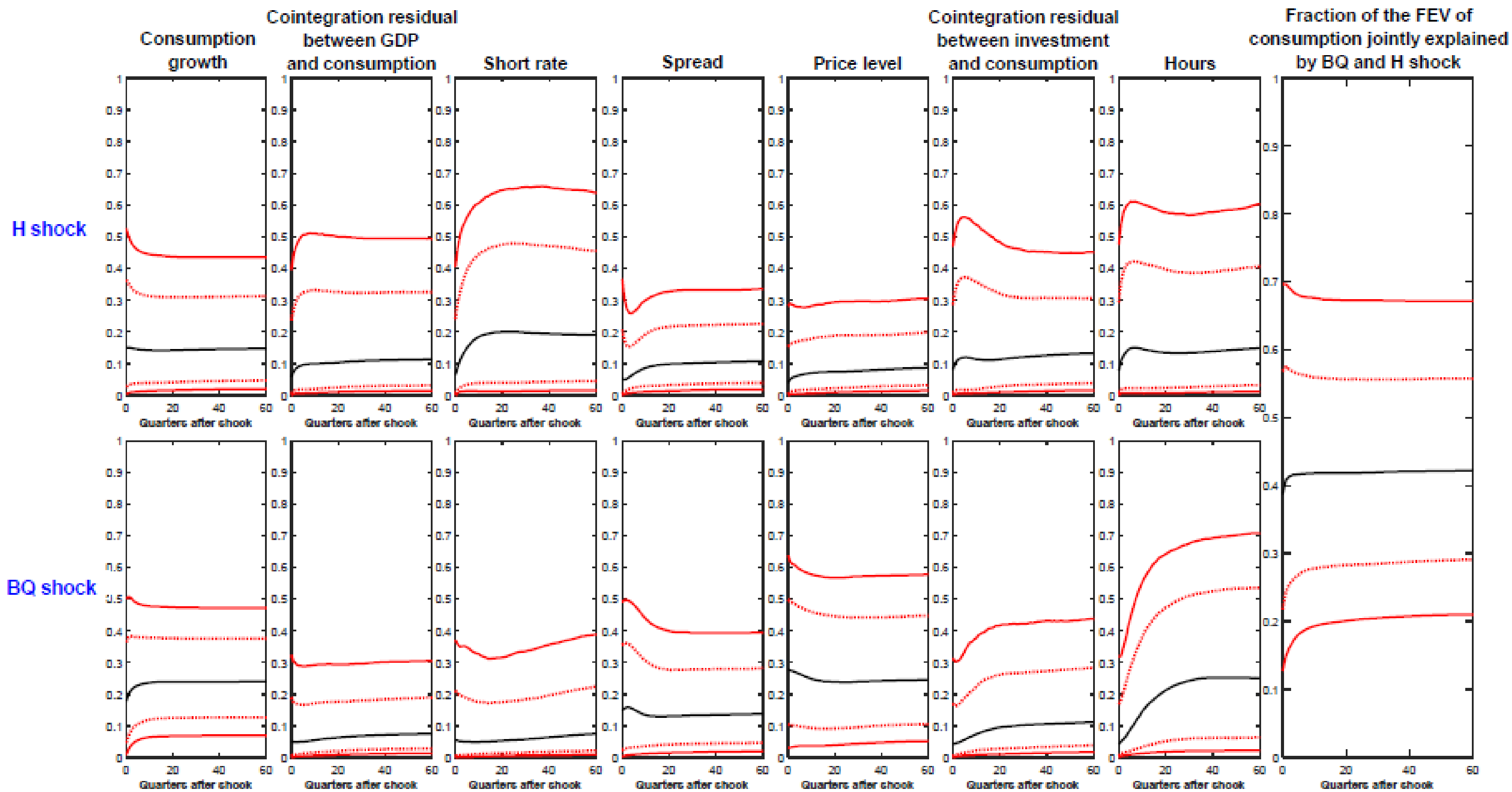

Figure A.11 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR (including the ZLB period)

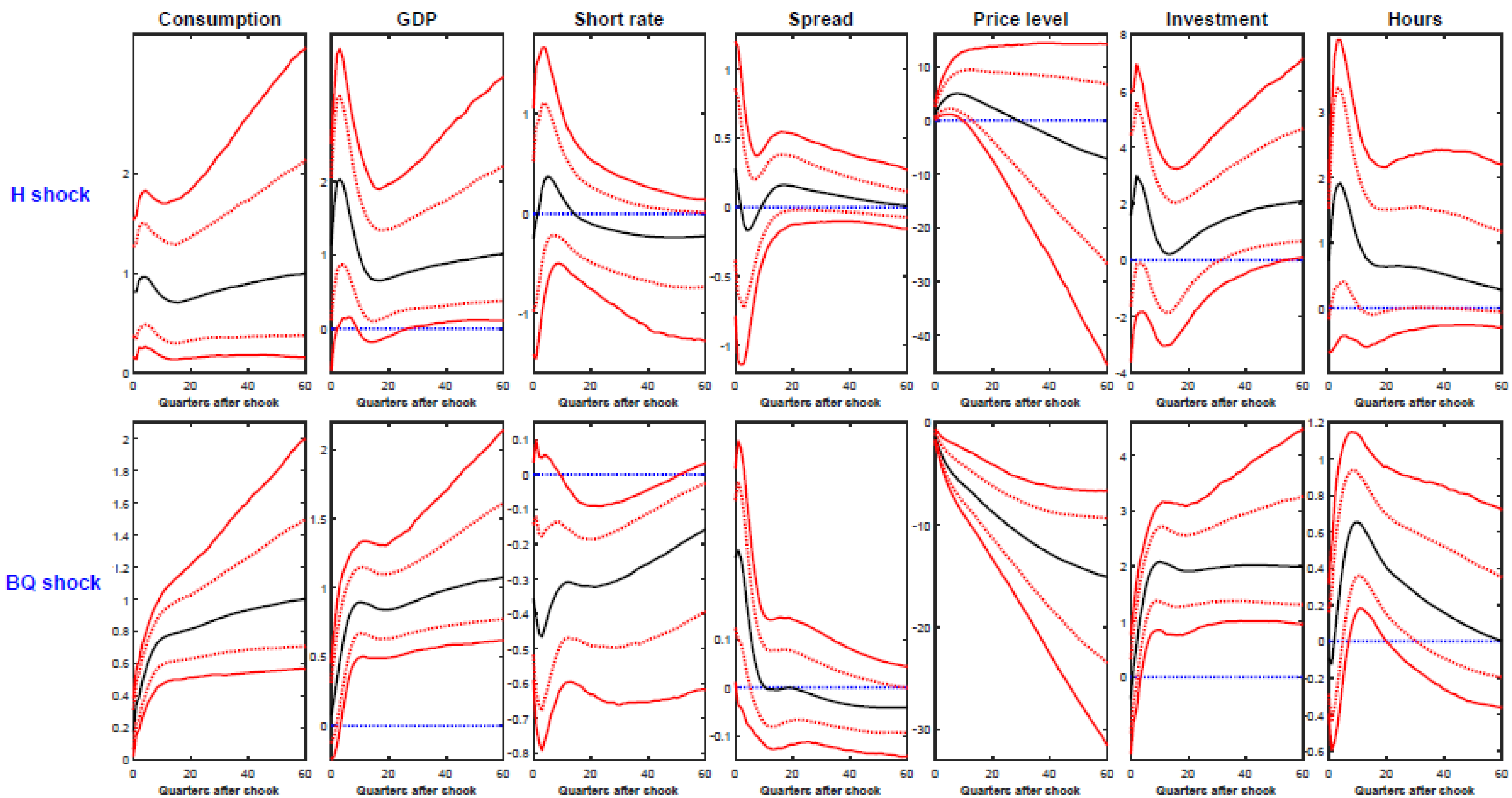

Figure A.12 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR (excluding the ZLB period)

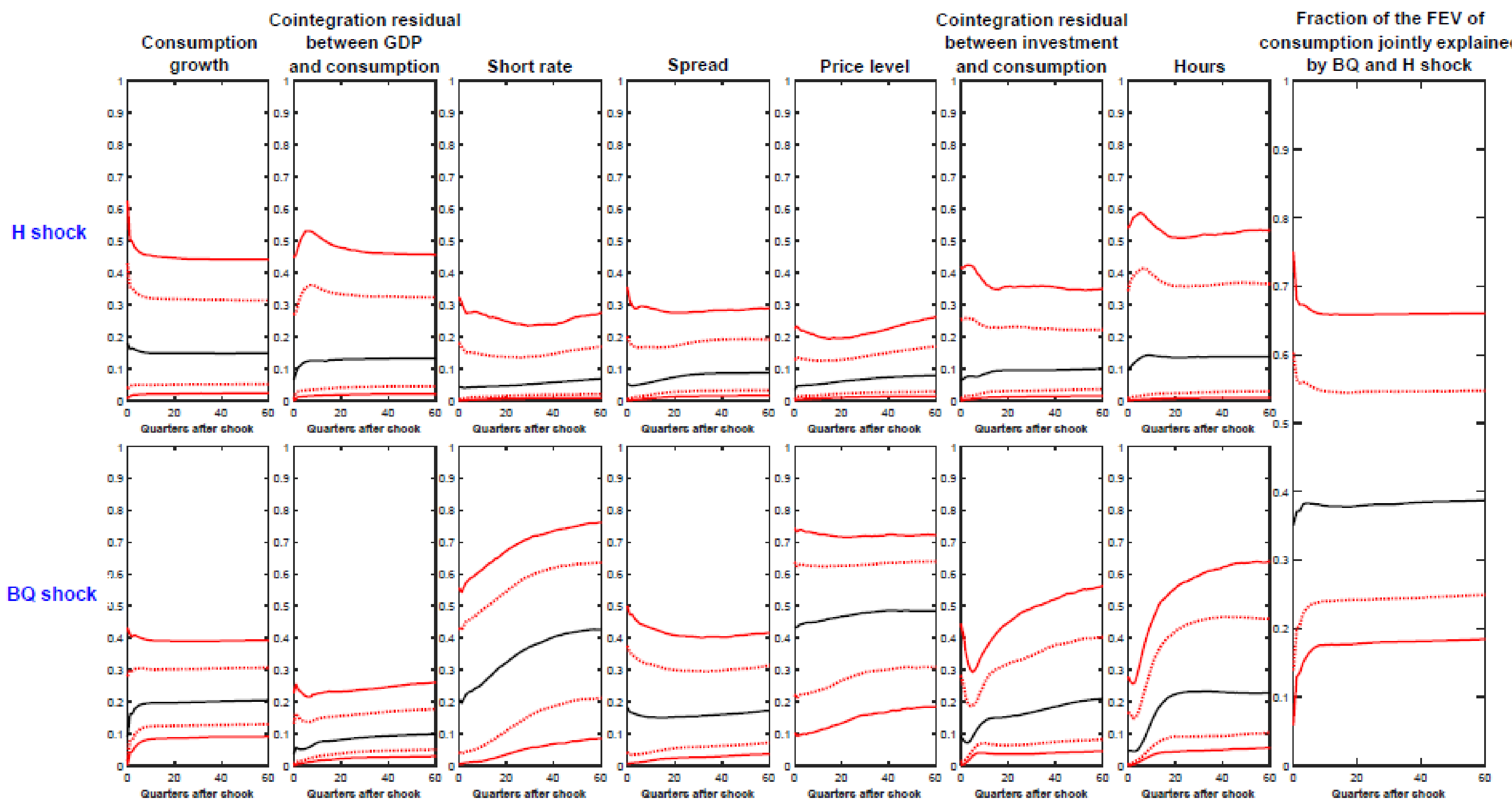

Figure A.13 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR (excluding the ZLB period)

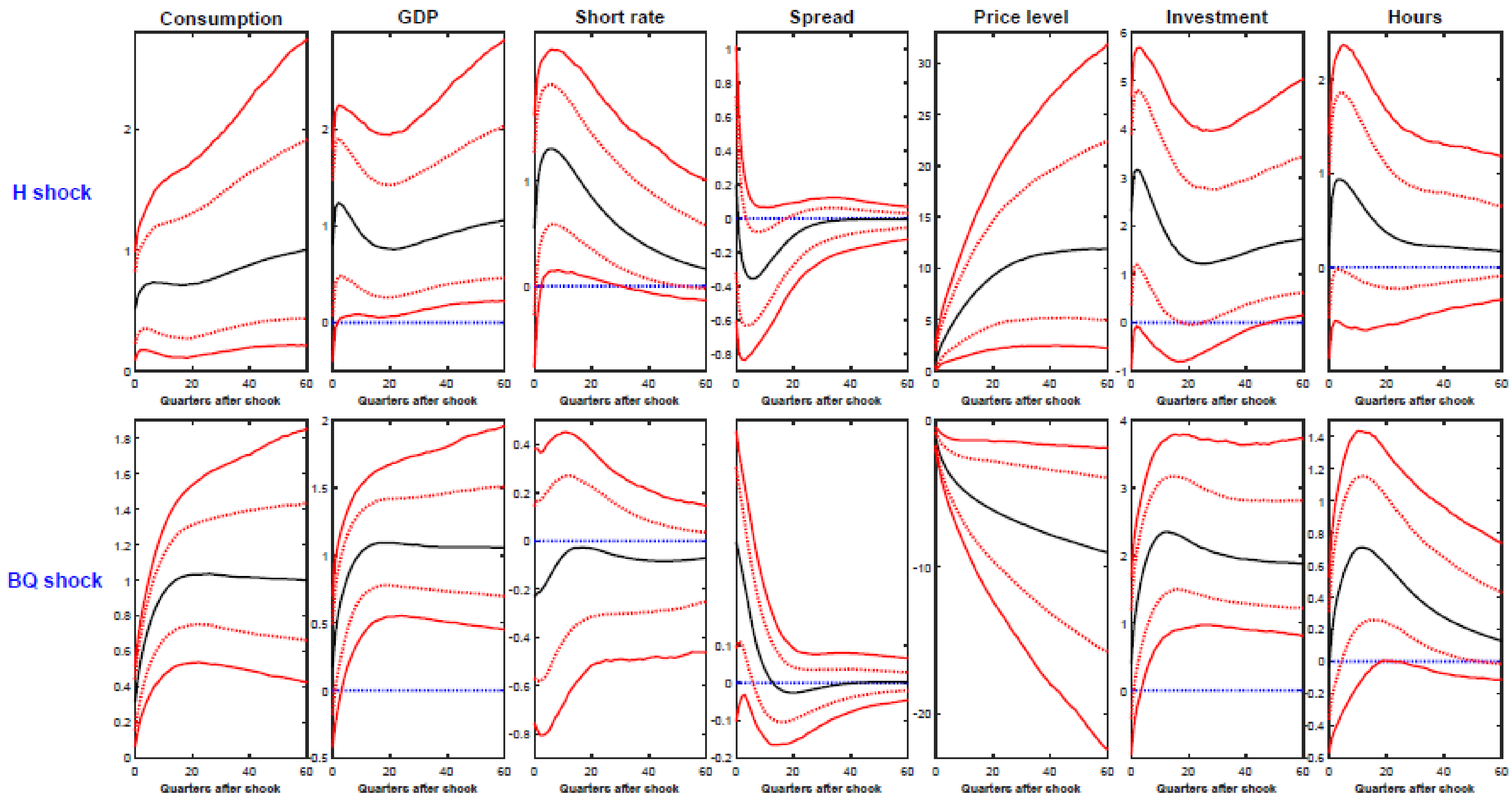

Figure A.14 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (including the ZLB period)

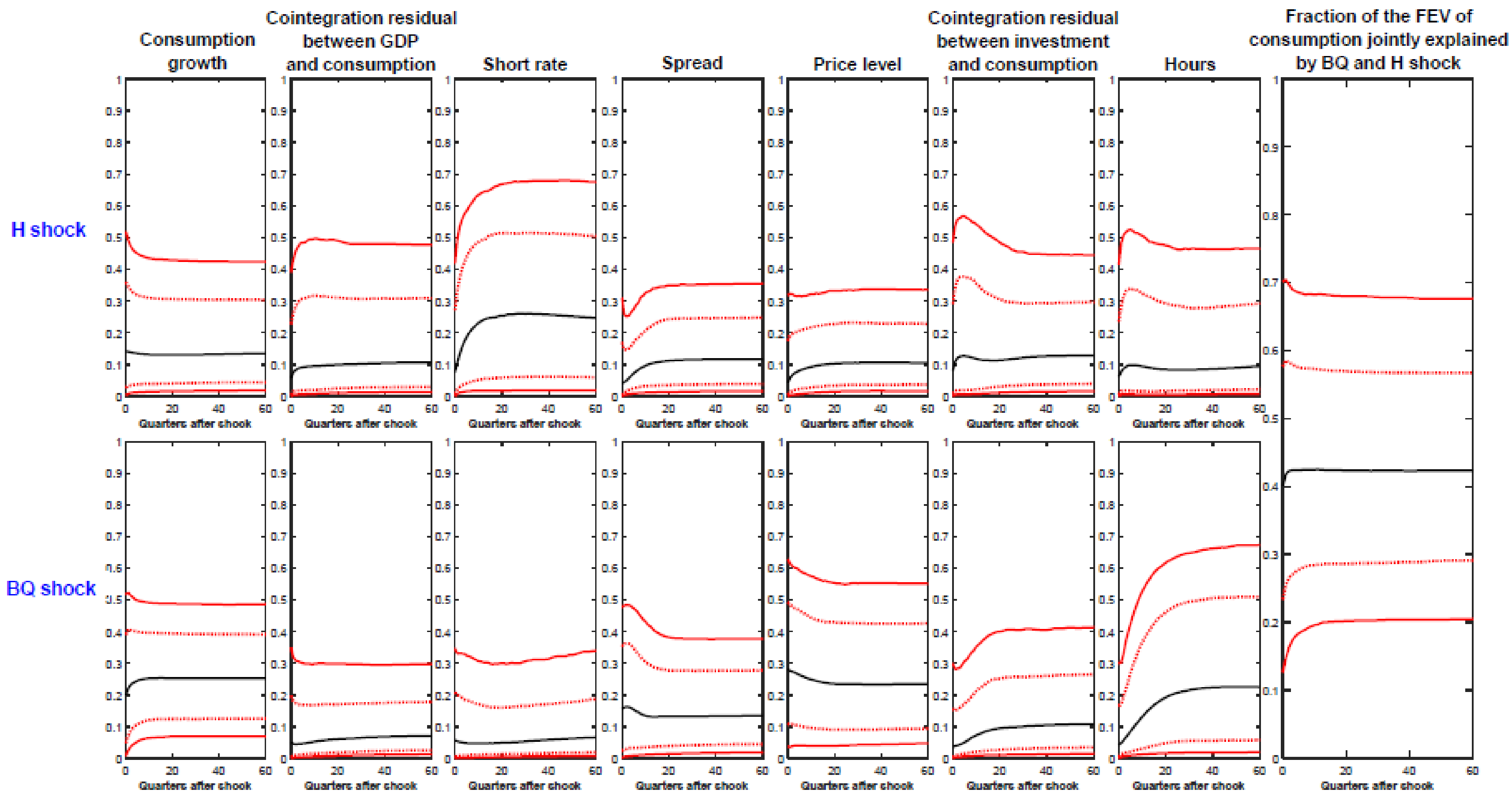

Figure A.15 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (including the ZLB period)

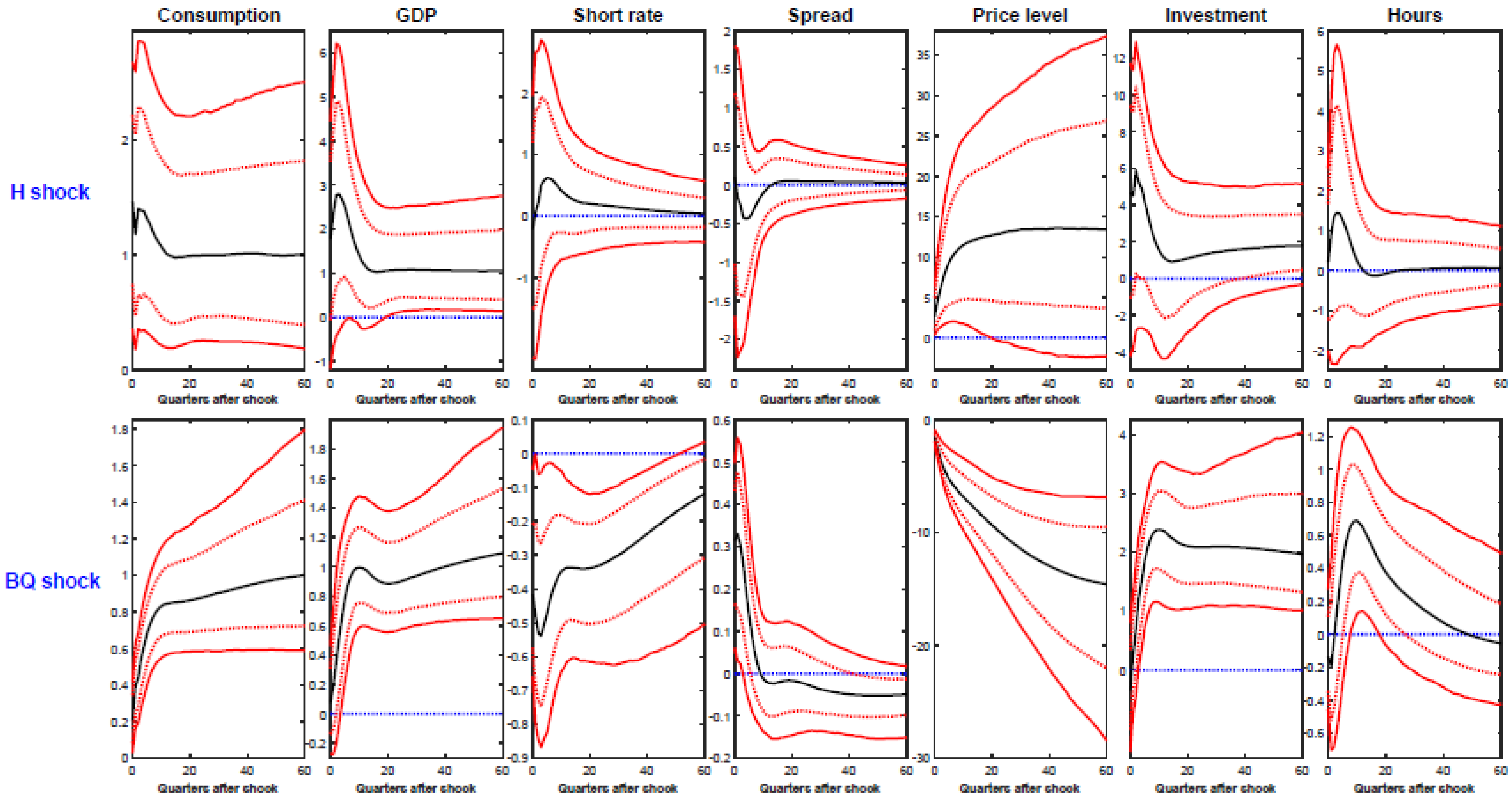

Figure A.16 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

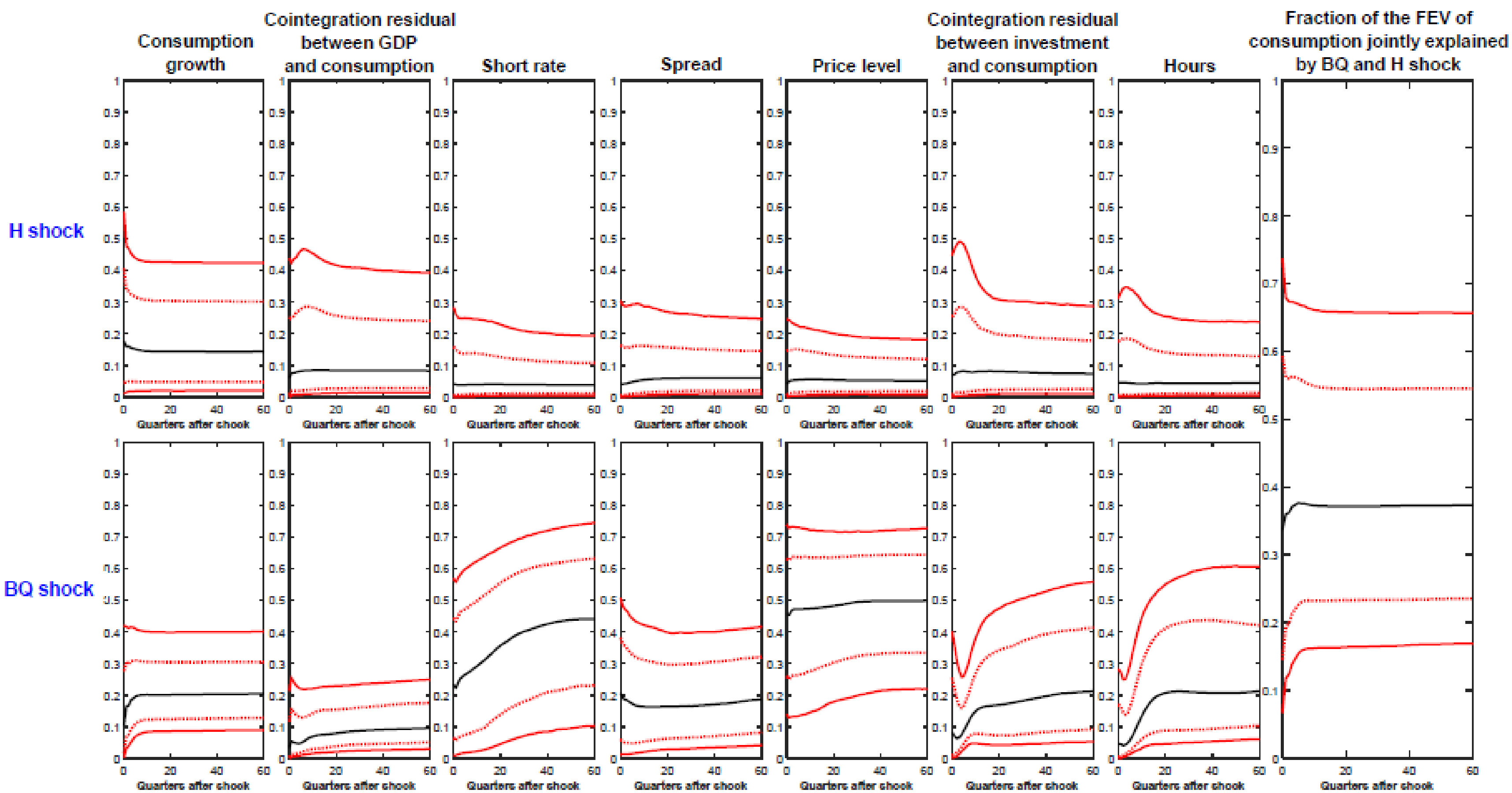

Figure A.17 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

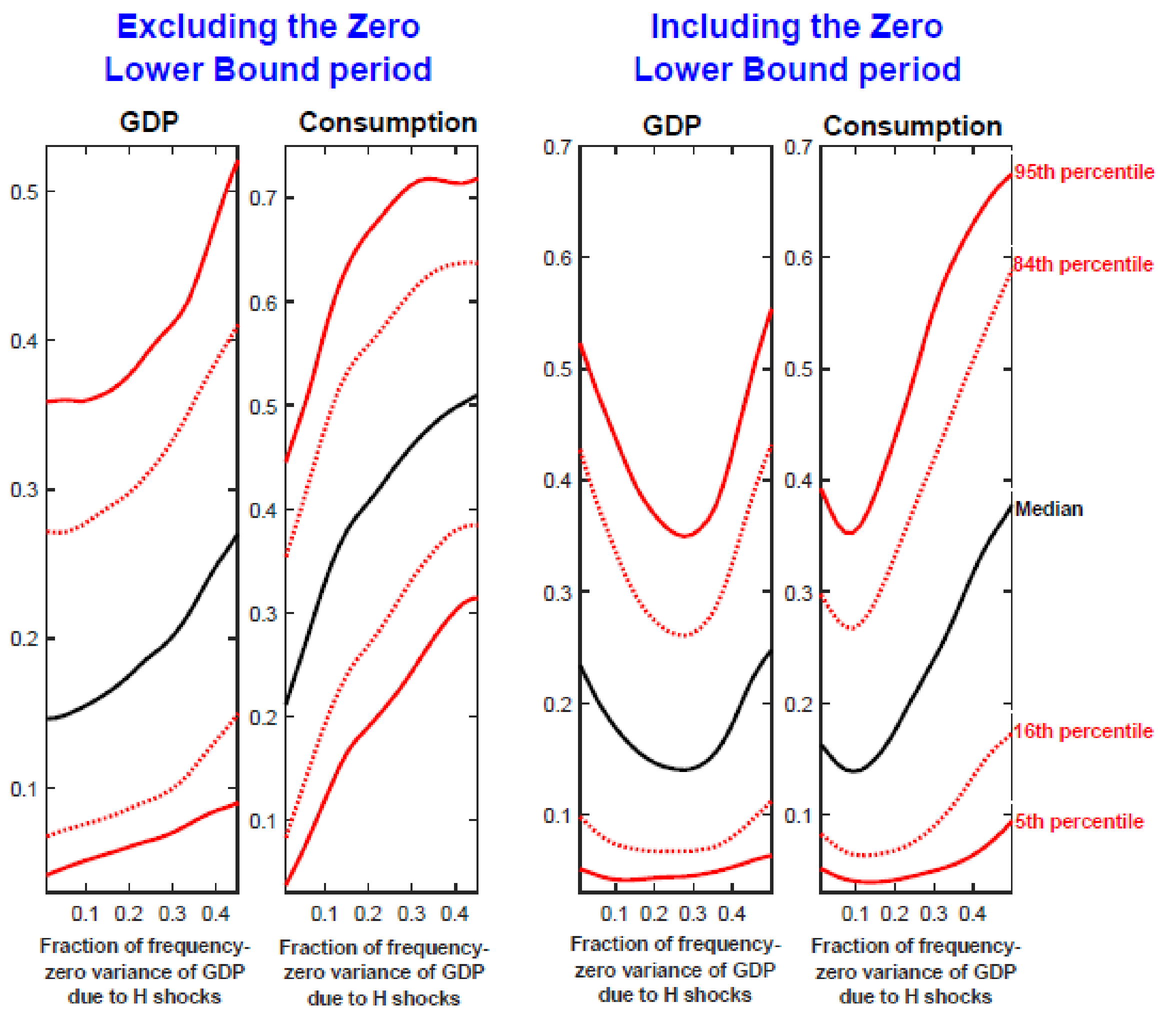

Figure A.18 Median and selected percentiles of the Monte Carlo distribution of the Kolgomorov-Smirnov statistic

Evidence on the Convergence of the Markov Chain

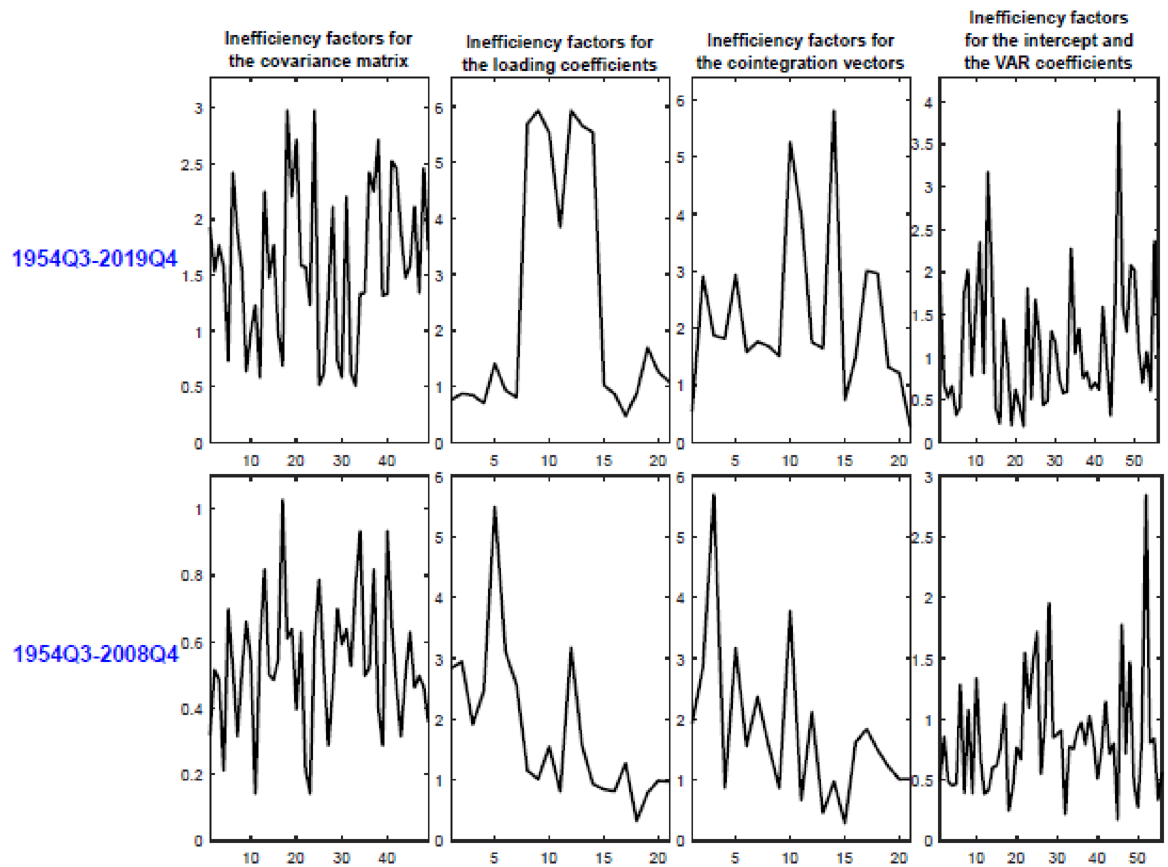

Figure A.19 Evidence on the convergence of the Markov chain for the 7-variables cointegrated VAR: Geweke's (1992) inefficiency factors of the draws for each individual parameter

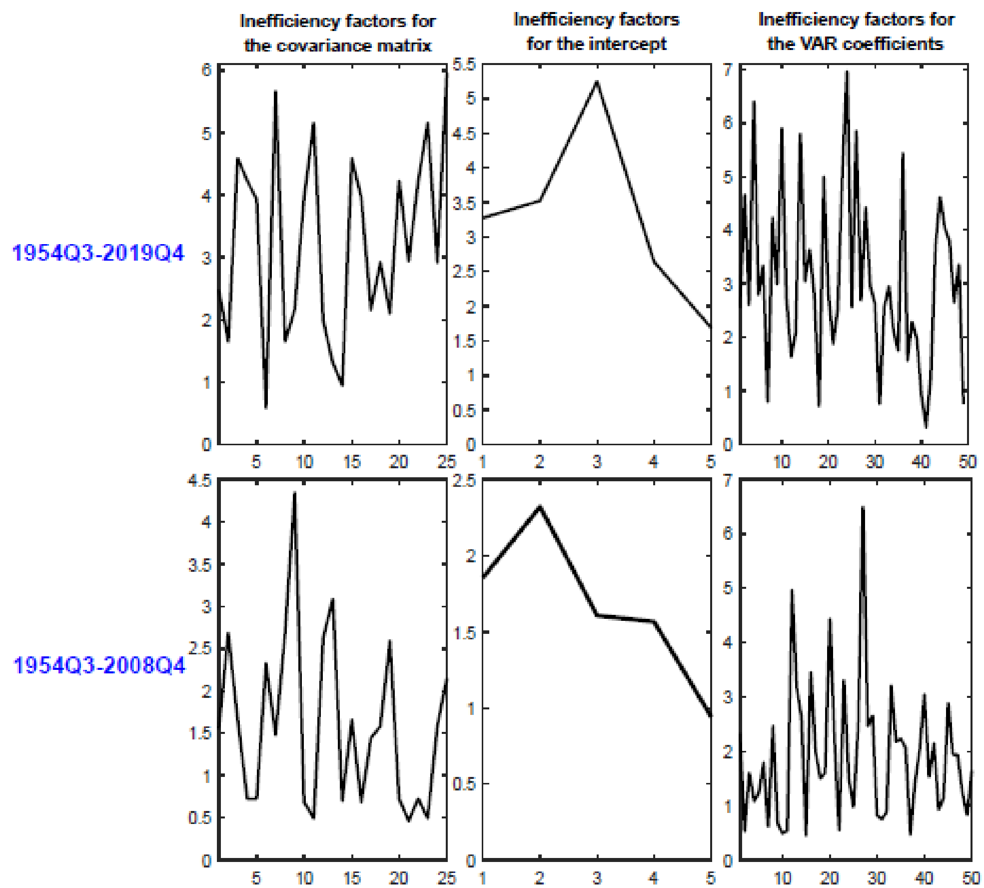

Figure A.20 Evidence on the autocorrelation of the draws for the 5-variables stationary Bayesian VAR: Geweke's (1992) inefficiency factors of the draws for each individual parameter